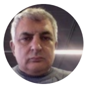
Application of Multivariate Analysis Techniques to Atmospheric Cherenkov Imaging data from the Crab Nebula

Article · July 1991



Ashot Chilingarian
Yerevan Physics Institute

1991ICRC....1..460C

Application of Multivariate Analysis Techniques to Atmospheric Cherenkov Imaging data from the Crab Nebula.

A. A. Chilingarian
Yerevan Physics Institute, Republic of Armenia.

M. F. Cawley
Physics Department, St. Patrick's College, Maynooth, Ireland.

Abstract

Using simple one-dimensional discrimination cuts on Cherenkov imaging data form the Crab Nebula, the Whipple Collaboration has established this object as a source of TeV gamma rays at a high level of statistical significance. Further gains in sensitivity should be possible through more sophisticated multivariate analysis of the Cherenkov images. In particular, we show that additional discrimination information is available in differences in correlations between image parameters associated with proton and gamma-ray induced showers. A new discrimination algorithm is described which improves the sensitivity relative to a non-imaging system by a factor of 6.6 and yields a 27 sigma effect on data from the Crab Nebula.

1. Introduction. Using the Cherenkov Imaging technique, the Whipple Collaboration has established the Crab Nebula to be a steady source of TeV gamma rays. In Weekes et al. (1989), a 9 sigma DC excess is reported from the source, while in Vacanti et al. (1991) an upgrade of the detector to a finer pixel resolution permitted a 20 sigma excess to be observed over a much shorter observation span. In both of these works, a relatively simple single dimensional discrimination analysis ('Azwidth') is used to eliminate (in the case of Vacanti et al.) 97% of the background while retaining an estimated 60% to 70% of the signal.

It has been proposed by various authors (eg. Zyskin and Kornienko, 1989; Aharonian et al. 1991) that standard multivariate data analysis techniques could be gainfully applied to the problem of partitioning the Cherenkov image feature space. A common element in many of these standard techniques is the exploitation of correlation information between features (as in multivariate distance metrics such as the Mahalanobis distance, for example). It is this element which distinguishes the Multivariate approaches from the simpler 1-D discriminant techniques, even when the latter are applied in sequential or 'majority logic' fashion as advocated by Hillas (1985) (in that paper, the author found on the basis of simulated data that sensitivity could be increased by demanding that at least 4 out of 6 1-D discriminants lie in the gamma domain).

2. Selection of an Optimal Feature Space. With the present Cherenkov Imaging detector on the 10m reflector at Mt. Hopkins (Cawley et al. 1990), only information regarding the angular distribution of the Cherenkov light is recorded; other possible discriminants such as pulse duration, UV content, etc. are not available for most of the data. A multidimensional feature space may therefore be constructed using parameters such as Width, Length, Miss, Distance, Azwidth, Compactness and Zone (Weekes et al. 1989) which distil information from the image regarding its orientation relative to the centre of the field and its angular extent. For completeness, we also tested additional image features: 'Skew', a measure of the image asymmetry; 'Alpha', the angle between the major axis and the line to the centre of the field; 'Elp', the image ellipticity; 'Pt', a parameter which judges the degree of pointing of the image,

taking into account the orientation of the 'tail' of the predominantly comet-shaped images (a tail pointing away from the centre is favoured by images from the direction of the source). We have used the prescription of Aharonian et al. (1991) to select the optimal feature combination from this list:

i) select the best individual image parameters on the basis of their 1-D discrimination performance,

ii) select the best pairs of parameters such that at least one of the parameters from (i) is included and such that their correlations are significantly different for signal and background events.

In addition to these criteria, it is important that the chosen feature space results in a tight clustering of the signal events, as this facilitates a greater degree of background rejection.This prescription was applied to two independent sets of simulated Cherenkov images (Aharonian et al. 1991; Macomb and Lamb, 1990). The degree of correlation difference between pairs of parameters for gamma rays and protons was gauged using the Fisher test - a high value of the Fisher statistic indicates a large difference in correlation behaviour. The Fisher matrix in table 1 for the simulations of Macomb and Lamb (1990) indicates high degrees of correlation differences for several pairs of features, eg. Skew and Length, Miss and Distance, Azwidth and Width. Scatter plots of gamma and proton images for each of these promising pairs of features were then examined to determine the extent of the clustering. It was found, for example, that the Azwidth-Width pair yield a tight cluster (fig. 1a) whereas the Miss-Distance cluster is broader (fig. 1c). Thus, even though the latter gives a higher correlation difference in table 1, the Azwidth-Width combination is found to perform better as a means of separating the two clusters. The Azwidth-Width correlation difference may be rationalised as follows: for gamma rays, we expect a strong correlation between these parameters because the direction of arrival of the gamma ray causes a radial alignment of patterns in the focal plane, and Azwidth practically equals Width. Images formed from the isotropically distributed protons have no preferable orientation, and thus there is no pronounced correlation between the two parameters. (Note that Azwidth must always be greater than or equal to Width, hence the absence of any points in the lower right of the scatter plots in fig. 1a,b). A point such as 'P' in fig. 1a can be rejected due to lack of correlation even though it would be accepted if sequential 1-D cuts on Width and Azwidth were performed (effectively accepting all events in a rectangular region close to the origin of the scatter plot).

	Skew	Dist	Alpha	Elp	Width	Len	Miss	Pt	Azwid
Skew	0.0								
Dist	9.8	0.0							
Alpha	10.3	36.3	0.0						
Elp	7.5	31.7	25.9	0.0					
Width	26.8	17.2	14.4	0.7	0.0				
Len	39.9	15.3	3.1	7.8	11.0	0.0			
Miss	4.7	50.0	5.3	10.3	7.5	5.0	0.0		
Pt	29.9	7.8	21.7	2.0	22.0	24.7	18.8	0.0	
Azwid	25.5	16.8	4.0	27.1	31.7	19.4	4.7	7.5	0.0

Table 1: Fisher Matrix for simulated gamma and proton images.

3. Construction of a Multivariate Discriminant using Correlation Information. Using the basic guidelines outlined above, a region was found in a 3-D feature space defined by Azwidth, Width and Length which gave a Q or sensitivity enhancement factor of 6.0 based on simulated events. We define Q as

$$Q = (N_g/N_{go})/(N_p/N_{po})^{\frac{1}{2}}$$

where N_g (N_p) is the number of gamma rays (protons) surviving the cut, N_{go} (N_{po}) is the number of gamma rays (protons) before application of any cut. This region utilises the strong correlation between Azwidth and Width for gamma rays, and introduces length to further reduce the background contamination. This 'wedge' shaped gamma-ray domain was then tested as a discriminant on the 1988/89 Crab Nebula data discussed in Vacanti et al. (1991). To allow for slight differences between real and simulated data, a range of wedge boundaries were tested - this discriminant is thus to some degree optimised on the data (about 50 trials). Furthermore, the wedge boundaries were allowed to vary for different Zones, where a zone is defined by the region in the image place where the maximum pixel signal occurs (Weekes et al. 1989). The optimal boundary was defined by:

i) Azwidth < 0.16 for zones 2,3
< 0.15 for zones 4,5
ii) Length < 0.3
iii) Width/Azwidth > 0.92.

Cuts (i) and (ii) are simple 1-D discriminants, while (iii) embodies the correlation information between Azwidth and Width. Events were eliminated if the maximum tube occured in zones 0,1, or 6, or if the width of an event was zero (compatible with a local muon signature). Results of the application of the Wedge discriminant are shown in table 2.

	ON	OFF	DIFF	DIFF/ON	SIGNIFICANCE
RAW	506255	501408	4847	0.96%	4.8 σ
WEDGE	6017	3381	2636	43.8%	27.2 σ

Table 2: Application of Wedge cut to Crab Nebula data.

4. Discussion. The Q factor estimated for the real data from table 2 is 6.6, which compared well with the sensitivity improvement predicted from simulations (note that Q only approximates the ratio of the significances - the Q value and significance ratio diverge when N_g becomes an appreciable fraction of N_p as is the case in table 2). An estimated 54% of the original gamma-ray signal is retained, while 99.3% of the background is rejected. Of the 65 28-min ON/OFF scans analysed, over 33% showed greater than 4 sigma excesses following application of the wedge discriminant; significant detection of the Crab Neubula in under one hour is therefore possible using this discrimination technique. It is clear that the boundaries of the wedge-shaped gamma domain are severely contrived in the interests of simplicity. We have attempted to determine the optimal shape of the gamma cluster from simulations using such approaches as nearest-neighbour non-parametric density estimation (using a multivariate distance metric such as the Mahalanobis distance to determine the number of gamma and proton neighbours around each simulated point in multidimensional feature space). Improved sensitivity was achieved in simulated data through such methods but this improvement did not transfer successfully to the real data. Further investigations are ongoing to determine the precise reason for this.

References.

Aharonian, F.A. et al. (1991) Nuclear Inst. and Methods, A, 302,522
Cawley, M.F. et al. (1990) Experimental Astronomy, 1,173
Hillas, A.M. (1985) Proc. 19th ICRC, LaJolla, 3, 445
Macomb, D.J. and Lamb, R.C. (1990) Proc. 21st ICRC, Adelaide, 2,435
Vacanti, G. et al. (1991) Ap. J. 377, 467

Weekes, T.C. et al. (1989) Ap. J. 342, 379

Zyskin, Yu. L. and Kornienko, A.P. (1989) Proc. Workshop on VHE Gamma-ray Astronomy, Crimea, 143

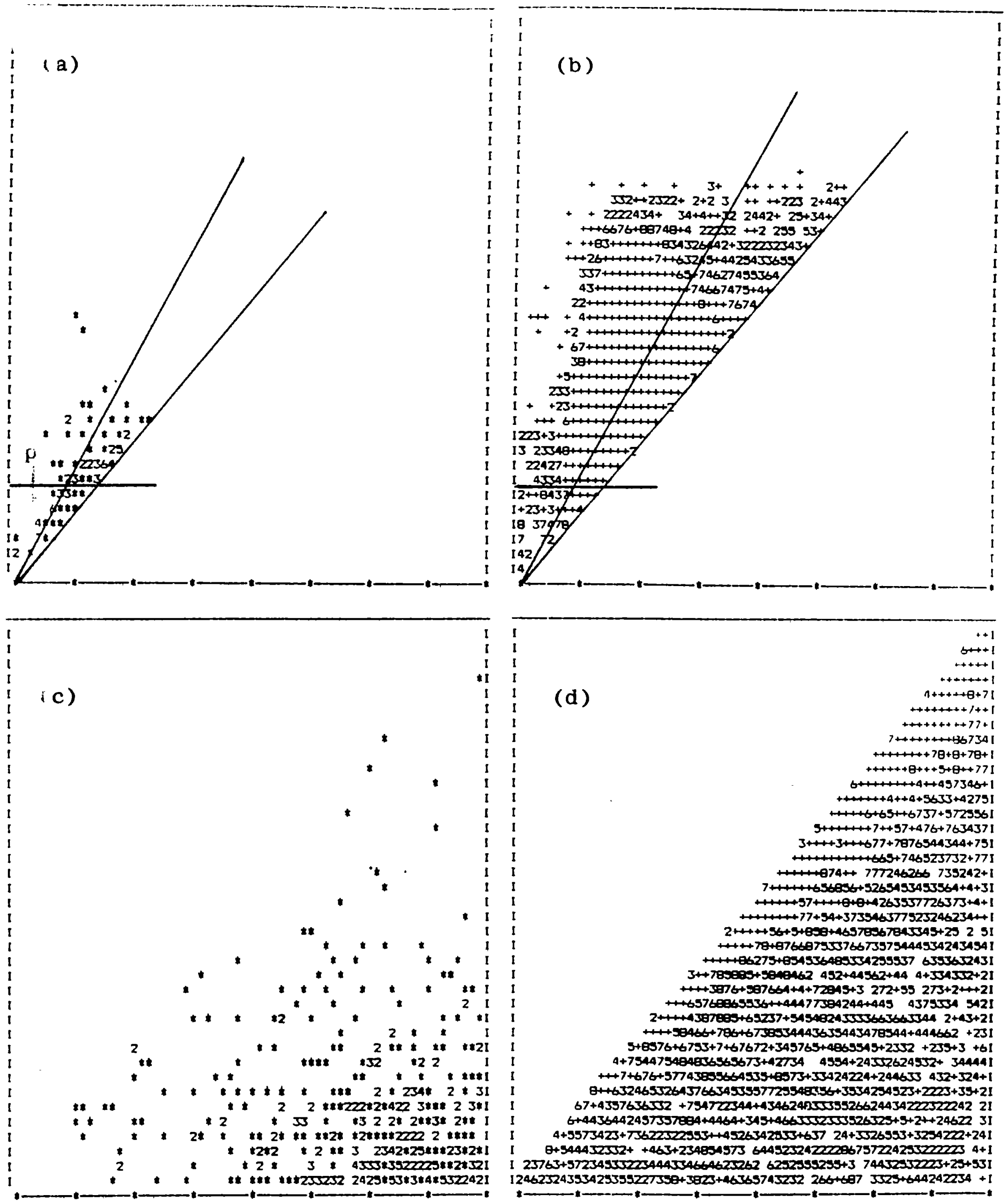


Fig. 1: a) Scatter plot of Azwidth against Width for gamma rays. Point p would be selected by a 1-D Azwidth cut (region below horizontal line) but not by a correlation cut (region bounded by two diagonal lines). b) Azwidth against Width for protons. c) Miss against Distance for gamma rays. d) Miss against distance for protons.