A Model Independent Method for Determination of Muon Density Fluctuations in EAS

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Abstract

It is shown how to determine fluctuations of the muon density in EAS at a given distance from the core, for showers with a fixed size. The method does not make use of any pre-assumed lateral muon distribution and uses only information whether muon detector has been hit by at least one muon.

1 Introduction:

The KASCADE experiment gives a unique opportunity to study in detail some shower characteristics due to a large number of detectors. In particular it is suitable to determine fluctuations of the muon densities in showers using the information from the Array of 192 muon detectors (3.24 m^2 each). In this paper we present how to determine the muon density probability distribution at a given core distance R for showers with fixed shower size N_e. In our method the muon density ρ_μ at a given R, is not being determined for each individual shower as that would need an a priori assumption about its lateral distribution. We prefer to avoid this and the fluctuations of $\rho_\mu(R)$ have been reconstructed from a sample of showers with fixed N_e.

2 The idea:

The shower sample used in this analysis has been obtained from a sample of the KASCADE data. We have chosen only almost vertical showers (zenith angle $< 18^o$). Our sample has been further divided into rather narrow bins of shower size $\Delta \log N_e = 0.1$. Our aim is to determine probability density distribution f(N) of the number of muons N falling on a fixed distance ring, for a sample of showers from a fixed N_e bin. The core distance has been divided into bins of $\Delta R = 10$ m. We shall use here the information from the muon Array detectors, with $E_{th} > 0.3$ GeV. Fluctuations of N are caused by fluctuations in shower development in the atmosphere and by the distribution of the primary particle mass.

Let us first choose the showers with a fixed number of muon detectors m at a given distance ring. If their number is $n(m)$ then the average number of showers $\langle \mathrm{F}(k;m) \rangle$ with k (out of m) muon detectors being hit by at least one muon, should be

$$\langle \mathrm{F}(k;m) \rangle = n(m) \int_0^\infty \binom{m}{k} \left(1 - e^{-\alpha N}\right)^k e^{-\alpha N(m-k)} \cdot \mathrm{f}(N) dN \qquad (1)$$

where $\alpha = S_{det}/S_{ring}$ (ratio of the area of a muon detector to that of the whole distance ring). In (1) we have assumed that showers have radial symmetry and that the N muons fall on the whole ring $(R, R + \Delta R)$

independently and randomly. To determine $\mathrm{F}(k;m)$ experimentally we need a criterion for a muon detector to be hit by at least one muon. First, we make our analysis only for distances $R > 40$ m, where the punch through effect can be almost neglected (at least for smaller showers). Next, after looking at distributions of the energy deposit for single muons from many muon detectors, we have chosen $E > 3.5$ MeV as our condition that a detector has been hit by muon(s). We would like to underline here that for our purpose we do not have to worry about how many muons have hit a detector (which is not always possible with a good accuracy).

The actually observed number of showers $\mathrm{F}(k;m)$ with k hit detectors fluctuates with the Poissonian distribution around its expected value, given by (1) and is of course the better representation of its mean $\langle \mathrm{F}(k;m)\rangle$, the bigger is the number of showers $n(m)$. The KASCADE experiment has a big advantage of having many muon detectors (192 in the Array), allowing the number m of available detectors in a given ring to reach values even above twenty (being around 10 most frequently).

Thus, in principle we can measure many values $\mathrm{F}(k;m)$ as $0 \leq k \leq m$, and for many m as well. Our sample, however, was not big enough for all experimental $\mathrm{F}(k;m)$ to represent their expected values $\langle \mathrm{F}(k;m)\rangle$ with a good accuracy. So, to determine $\mathrm{f}(N)$ (for any N_e and R bin) we have summed our $\mathrm{F}(k;m)$ histograms over all m (over all positions of the shower core), obtaining histograms $\mathrm{F}(k) = \Sigma_m \mathrm{F}(k;m)$. By summing up over m we lose some information contained in the k distributions for each individual m. We gain however, by getting smaller statistical relative uncertainties of $\mathrm{F}(k)$ and by simplifying evaluation of $\mathrm{f}(N)$.

3 Factorial moments of the distribution of k and a check of the Array:

From (1) are can easily calculate moments of the probability distribution of k: $\langle k\rangle$, $\langle k^2\rangle$ and so on. It turns out, however, that in this case it is the factorial moments of k which are in a simpler way related to the muon number distribution $\mathrm{f}(N)$:

$$\langle k(k-1)\dots(k-i+1)\rangle = m(m-1)\dots(m-i+1)\cdot\int_0^\infty \left(1-e^{-\alpha N}\right)^i \mathrm{f}(N)dN \qquad (2)$$

for $i = 1, 2, \dots$. As $1 - e^{-\alpha N} = p$, where p is the probability of hitting a detector once N muons have fallen on the ring, we see that the integrals in the right-hand sides of (2) represent the successive moments of the distribution of p. Thus, in principle, having all moments one could obtain the probability distribution of p, $\mathrm{g}(p)$, and then $\mathrm{f}(N) = \mathrm{g}[p(N)]$. We notice, however, that the higher is the order of the factorial moment of k, the smaller is the part of the k distribution on which it depends. Thus, as the number of showers with higher k finally decreases, one would need very big statistics in order to determine higher order moments of k (and p) with a reasonable accuracy. So, in our analysis we shall not use formulae (2) to determine muon fluctuations $\mathrm{f}(N)$. We shall use them, however, to check the homogeneity of the detection conditions of the Array. From (2) it follows that neither $\langle k\rangle/m$ nor $\langle k(k-1)\rangle/m(m-1)$ should on average, depend on m, that is, on the position of the shower core. Fig.1 represents the experimentally obtained ratios, as a function of m, for fixed N_e and different R. It can be seen that, within the statistical errors, the ratios do remain independent of m for almost any case, confirming the homogeneity of the Array.

4 Methods of determining muon density fluctuations $\mathrm{f}(N)$:

As we have already explained, the basis for determining $\mathrm{f}(N)$ (for any fixed N_e and R bin) is a set of equations (1) summed over m, for $k = 0, 1..., m_{max}$. To find $\mathrm{f}(N)$, we have applied the three following methods:

4.1 Numerical fit: The integral in the right-hand side of (1) summed over m was approximated by a sum of 10 values of the integrated function at 10 values of N. The ten unknowns $\mathrm{f}(N_i)$ were then found by a maximum likelihood method allowing for the statistical fluctuations of $\mathrm{F}(k)$. The CERN program MINUIT was used to find these best fitting values $\mathrm{f}(N_i)$ on condition that $\mathrm{f}(N_i) \geq 0$.

4.2 Method using three moments of N distribution: It can be applied if $\alpha N \ll 1$. After expanding $e^{-\alpha N}$ in (2), keeping only first three terms and averaging over m we can express the three experimentally determined factorial ratios: $\langle k \rangle / \langle m \rangle$, $\langle k(k-1) \rangle / \langle m(m-1) \rangle$ and $\langle k(k-1)(k-2) \rangle / \langle m(m-1)(m-2) \rangle$ as linear combinations of $\overline{N}$, $\overline{N^2}$, and $\overline{N^3}$. The latter moments can be easily found. Next we assume that $\mathrm{f}(N)$ has a shape of a gamma function: $\mathrm{f}(N) \sim N^{p-1} e^{-qN}$, and calculate p and q from $\overline{N}$, and $\overline{N^2}$.

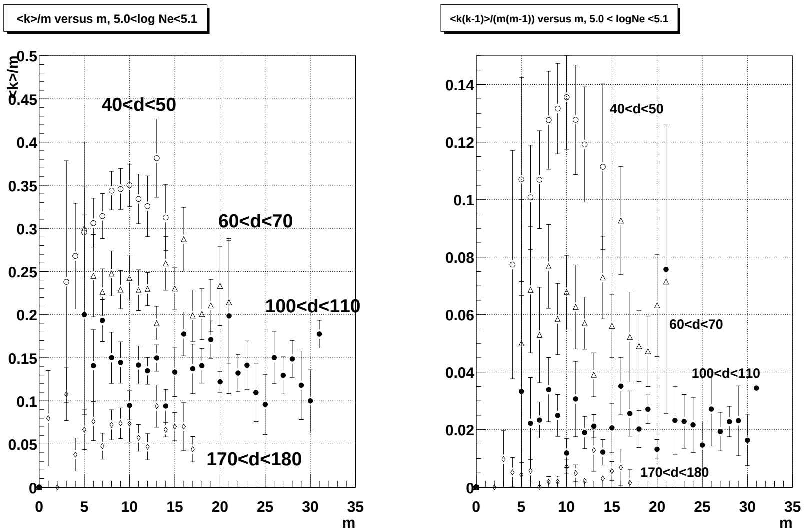

Figure 1: Ratios of the first (left) and second (right) factorial moments, $\langle k \rangle / m$ and $\langle k(k-1) \rangle / m(m-1)$ for $5 < \log N_e < 5.1$ and several $R(m)$.

4.3 Method adopting $\mathrm{f}(N)$ as a gamma function:

Here, we assume from the very beginning that $\mathrm{f}(N)$ can be described by a gamma function. Inserting it into equations (2a) and (2b) and averaging over m we can express the first two factorial ratios (as in the previous case) as function of p and q. The parameters p and q can be found numerically. This method does not require small muon densities, as in the case 4.2.

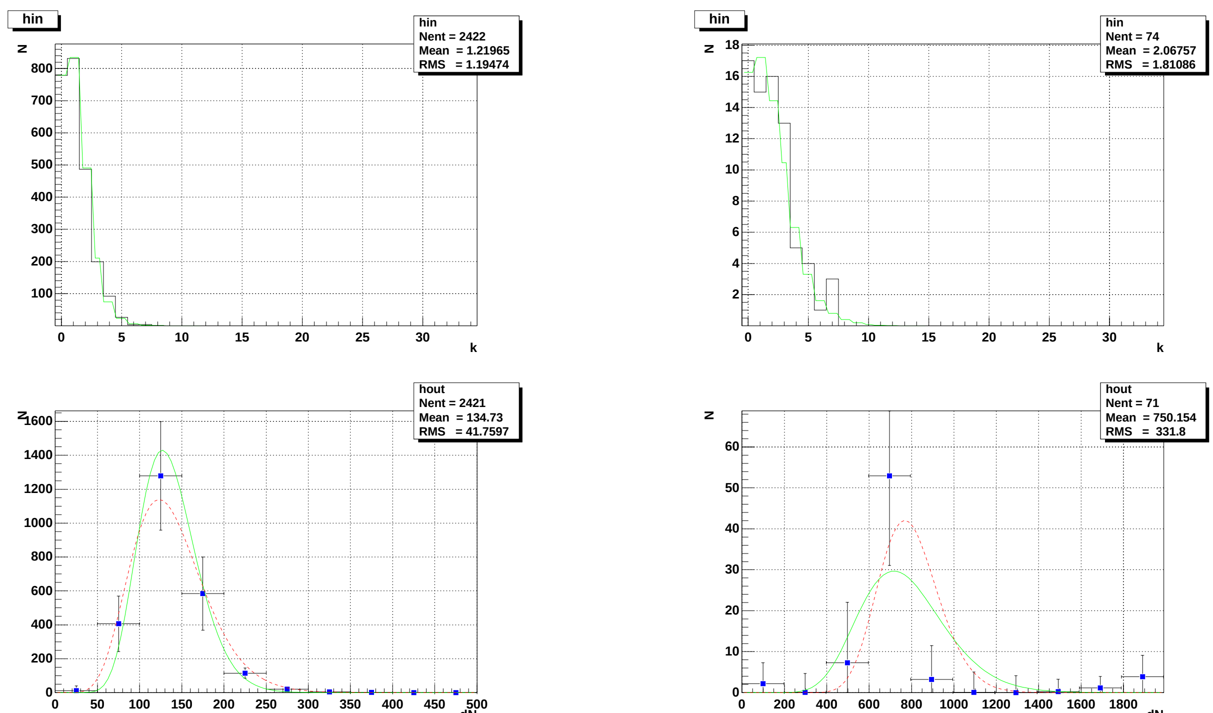

Figure 2: Distribution of the number of hit muon detectors (N_e, R fixed) – upper histograms. Corresponding calculated distributions of N – number of muons in the distance ring – lower graphs. Left graphs: $4.5 < \log N_e < 4.6$ and $60 < R < 70$m; right graphs: $5.5 < \log N_e < 5.6$ and $160 < R < 170$m. Points – method 4.1; dashed line – method 4.2; dotted line – method 4.3. The dotted histograms (upper graphs) are calculated $\langle \mathrm{F}(k) \rangle$ for $\mathrm{f}(N)$ found by method 4.1 (MINUIT).

5 Results:

Fig.2 illustrates the results of our analysis. The upper histograms are the observed distributions $F(k)$, chosen for some particular values of N_e and R. The lower graphs represent the corresponding distributions $f(N) \cdot \Delta N$ (multiplied by the total number of events for each case), obtained by the three methods. It can be seen that $f(N)$ obtained by the three different methods give similar results. For example for $5.0 < \log N_e < 5.1$ the differences of $\overline{N}$ calculated by the three methods are typically below 5%. The muon lateral distributions $\rho_\mu(R)$ obtained in this paper agree reasonably well with the results of another analysis of the KASCADE data (Leibrock et al., 1998), as it is shown in Fig.3. The dispersions of the distribution $f(N)$ obtained by the three methods differ more significantly, sometimes even by factor of two. The first method, based on MINUIT, gives usually the biggest value. As it fits ten values of $f(N)$, instead of two parameters of an analytic (gamma) function, as in the other two methods, we think that it gives a better description of reality (although it probably is more sensitive to fluctuations of $F(k)$. The dispersions σ_ρ relative to $\overline{\rho_\mu}$ determined by the first method, are presented in Fig. 4. We can see that typical values are 30 - 40%. For rather small N_e, where our statistics were the best, a trend of increasing relative fluctuations with the core distance R can be observed, although its statistical significance is not big. The fluctuations of ρ_μ for fixed N_e should be sensitive to the primary composition. Shower simulations are needed to show how big this effect is for the low energy muons considered here. With big statistics, available from the KASCADE experiment, it would be possible to obtain more detailed determination of the shapes of the N $(i.e.\ \rho_\mu)$ distributions.

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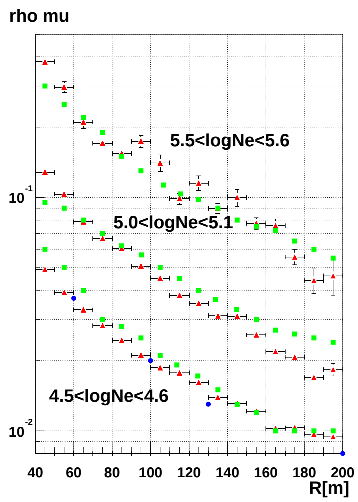

Figure 3: Comparison of the average lateral distributions of muons (ρ_μ in m^{-2}) obtained in this work (triangles) for three bins in N_e, with Leibrock et al. (squares). Circles show values obtained by EAS–TOP (Aglietta et al.) for $E_\mu > 1$ GeV.

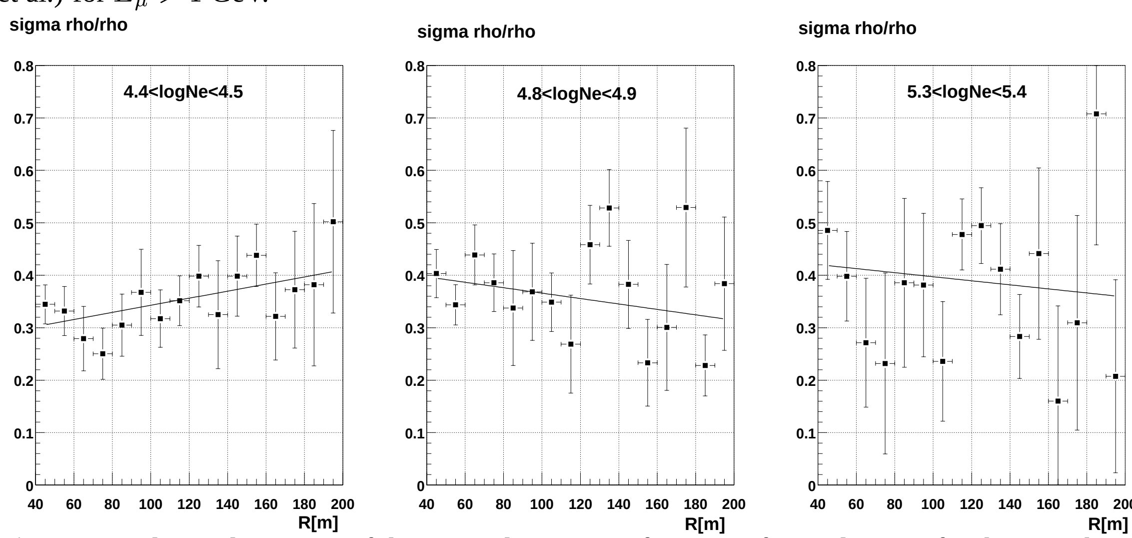

Figure 4: Relative dispersion of the muon density as a function of core distance for three N_e bins.

References

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