SIMULATION OF GPS CARRIER PHASE OBSERVABLES AND FEASIBLE ATTITUDE DETERMINATION ALGORITHMS FOR UNIVERSITETSKIY-TATYANA SATELLITE

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Precise attitude determination

- Precise attitude determination has an application in navigation, geodesy, gravimetry, magnetometry, construction and so on

- Carrier phase measurements provided by GPS hardware have potential accuracy of millimeters in linear distance and several advantages compared to inertial navigation systems; this allows using them for attitude determination

- Some special properties of carrier phase measurements require a stack of different mathematical methods and computational techniques to be processed instead of one algorithm

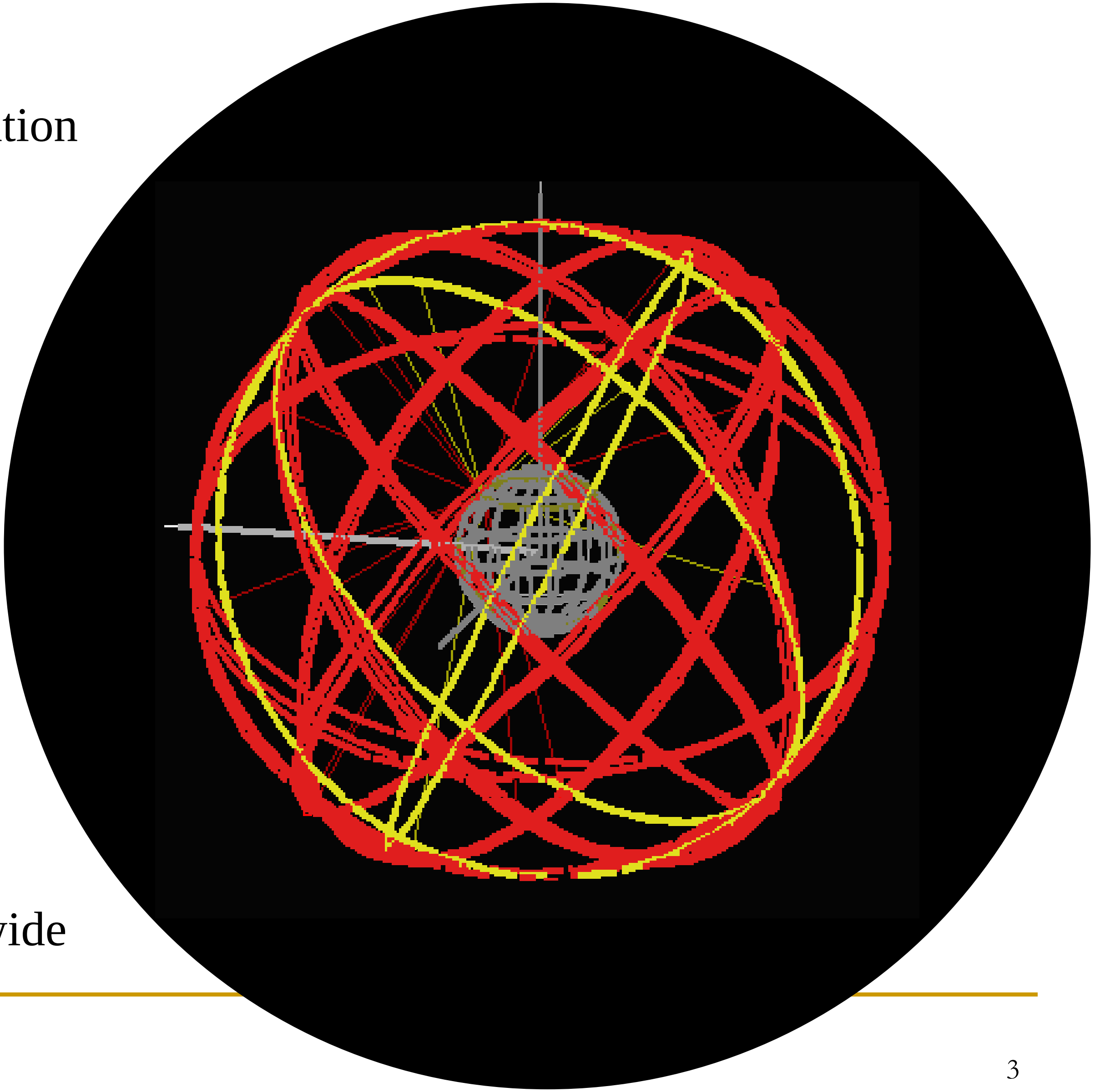
- Observability and consistency would be checked and evaluated at every stage of calculations

- Different methods and its effectiveness could be objectively compared using actual survey data

- Precisely known distances between antennas are available as an additional a priori information that could be involved in computational process

Satellite navigation systems

- 24-hour all-weather operation
- Common standards
- Up to 14 visible GPS satellites
- Up to 5 visible GLONASS satellites
- Redundant measurements
- Several carrier frequencies
- Emitted radio-signals are available to receive worldwide



Carrier phase measurements

- Commonly used carrier phase measurements equation (*n* is a number of visible satellites, *m* is number of antennas, N is time span in epochs):

$$\frac{\rho_{ij}(t_k)}{\lambda_{L_p}} = Z_{ij}^{L_p}(t_k) + \eta_{ij}^{p} = \varphi_{ij}^{p,sat}(t_k) + \eta_{ij}^{p} - \varphi_{j}^{p,rec}(t_k) +$$

$$+\Delta\varphi_{i}^{p,ion}(t_k) + \Delta\varphi_{i}^{p,trop}(t_k) + \Delta\varphi_{i}^{p,sat}(t_k) + \Delta\varphi_{j}^{p,rec}(t_k) + \Delta\varphi_{ij}^{p,mp}(t_k) + \Delta\varphi_{ij}^{p,s}(t_k)$$

$$\eta \in \mathbb{Z},\ \varphi_{i,j}^{sat,rec} \in [0,1]\ //\ \textit{Размерность} - \textit{циклы}$$

$$i = 1..n,\ j = 1..m,\ k = 1..N,\ p - \textit{номер частоты несущей}$$

p is carrier frequency number, all items are measured in cycles of a carrier

The wanted quantity is

$$\varphi_{ij}^{sat}(t_k) + \eta - \varphi_{j}^{rec}(t_k)$$

The rest of components are errors and instrumental noise

Single- and double-differencing

- Single-differencing cancels errors due to radio-signal propagation through ionosphere and troposphere, satellite clock errors:

$$\Delta Z_{ij}^{L_p}(t_k) = Z_{ij}^{L_p}(t_k) - Z_{ib}^{L_p}(t_k) =$$

$$= (\varphi_{ij}^{p,sat}(t_k) - \varphi_{ib}^{p,sat}(t_k)) - (\varphi_{j}^{p,rec}(t_k) - \varphi_{b}^{p,rec}(t_k)) + \Delta'\varphi_{j}^{p,rec}(t_k) + \Delta'\varphi_{ij}^{p,mp}(t_k) + \Delta'\varphi_{ij}^{p,s}(t_k)$$

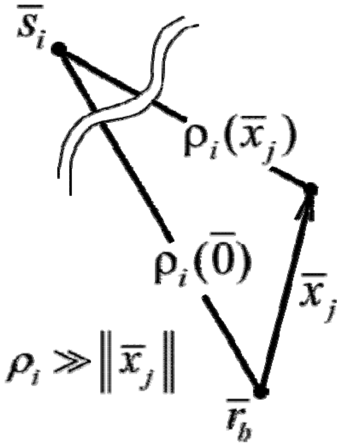
- Double-differencing cancels errors due to receiver clock errors; remaining irremovable errors are of less magnitude than others:

$$z_{ij}^{p}(t_k) = \Delta Z_{ij}^{L_p}(t_k) - \Delta Z_{zj}^{L_p}(t_k) =$$

$$= (\varphi_{ij}^{p,sat}(t_k) - \varphi_{ib}^{p,sat}(t_k)) - (\varphi_{zj}^{p,sat}(t_k) - \varphi_{zb}^{p,sat}(t_k)) + \Delta''\varphi_{ij}^{p,mp}(t_k) + \Delta''\varphi_{ij}^{p,s}(t_k)$$

- Number of measurements is reduced by 2

Attitude determination by processing carrier phase measurements characterization (page 1)



$$\rho_{ij}(\bar{x}_j) = \sqrt{(\bar{s}_i - \bar{r}_b - \bar{x}_j)^T (\bar{s}_i - \bar{r}_b - \bar{x}_j)} =$$

$$= \rho_{ij}(0) + \left[\frac{\partial \rho_{ij}(0)}{\partial x_j} \right]^T x_j + x_j^T \left[\frac{\partial^2 \rho_{ij}(0)}{\partial x_j^2} \right] x_j + \ldots = \sum_{l=0}^{\infty} A_l(x_j)$$

$$A_l(\bar{x}_j) \sim \left(\frac{\|\bar{x}_j\|}{\|\bar{s}_i - \bar{r}_b\|} \right)^{l-1} \|x_j\|$$

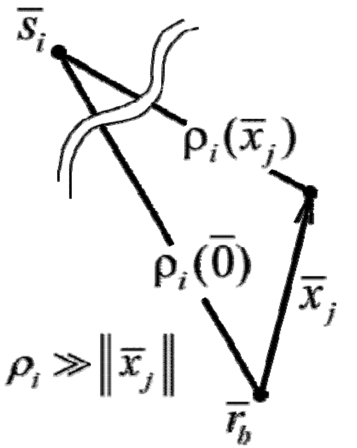
- The matter of attitude determination is relative positioning of several antennas (not less than 3)

- Differencing is quite natural: the equation of measurement $z = \Xi(\eta, x)$ is nearly linear

- Precise absolute positioning is not necessary

- Additional information (distances between antennas) is available to use

Attitude determination by processing carrier phase measurements characterization (page 2)



$$\rho_i(\bar{x}_j) - \rho_i(\bar{0}) = \lambda_{L_p}(Z_{ij}^{L_p}(t_k) - Z_{ib}^{L_p}(t_k) + \eta_{ij}^p - \eta_{ib}^p) =$$

$$= \left[\frac{\partial \rho_i(0)}{\partial \bar{x}_j}\right]^T \bar{x}_j + O\left(\frac{\|x_j\|^2}{\|s_{ik} - r_{bk}\|}\right)$$

$$\bar{x}_j,\ \bar{s}_i,\ \bar{r}_b \in \square^3,\ \frac{\|\bar{x}_j\|}{\|\bar{s}_i - \bar{r}_b\|} < 10^{-6},\ \textit{при}\ \ \|\bar{x}_j\| \square\ 10\textit{м},\ \|\bar{s}_i - \bar{r}_b\| \square\ 2 \cdot 10^7\ \textit{м}$$

- When antennas are close to each other (in dozens of meters) errors due to signal propagation through ionosphere and troposphere, linearization and clock errors are completely canceled by double-differencing

- Integer ambiguities introduce a significant complexity in estimation; the estimated vector is not observable at every particular epoch; observability is possible when a set of measurements for several epochs is available

Formal statements for different approaches to attitude determination by processing carrier phase measurements (page 1)

- Weighted least-squares method (errors in measurements are correlated)

$$\frac{1}{2}(z-H\xi)^T W(z-H\xi)\rightarrow\min_{\xi},\ \xi=\left[\eta:x\right]$$

$$\tilde{\xi}=(H^T H)^{-1}H^T z,$$

$$M[(\Delta\tilde{\xi})\cdot(\Delta\tilde{\xi})^T]=(H^T WH)^{-1}H^T WRW^T H(H^T W^T H)^{-1}$$

$$W=\sigma_0^2 R^{-1}\Rightarrow M[(\Delta\tilde{\xi})\cdot(\Delta\tilde{\xi})^T]=(H^T R^{-1}H)^{-1}$$

- Least-squares method with special restrictions given by known distances between antennas; certain algorithms deliver the solution

$$\frac{1}{2}(z-H\xi)^T(z-H\xi)\rightarrow\min_{\xi},\ \xi=\left[\eta:x\right],\ \xi\in\Xi,$$

$$\Xi=\{\xi\ \vdots\ \xi^T B_i^T B_i\xi=b_i^2\},\ i=1,\ldots$$

Решение доставляется известными алгоритмами

Formal statements for different approaches to attitude determination by processing carrier phase measurements (page 2)

- Reduction to static case using additional velocity measurements:

$$\dot{x}' = v',\ x'(0) = x'_0,\ x - x' = \chi$$

$$\dot{\eta} = 0,\ \dot{\chi} = \delta v = q$$

$$z - Hx' = H\chi - \eta + \rho,$$

$$\tilde{\chi} = L[z - Hx'],\ \tilde{x} = \tilde{x}' + \tilde{\chi}$$

- Reduction to static case and introducing the additional information as extra-measurements:

$$\dot{x}' = v',\ x'(0) = x'_0,\ x - x' = \chi,\ x^T B_i^T B_i x = b_i^2$$

$$\dot{\eta} = 0,\ \dot{\chi} = \delta v = q,\ (x' + \chi)^T B_i^T B_i (x' + \chi) = b_i^2$$

$$z - Hx' = H\chi - \eta + \rho,\ b_i^2 - \beta_i(x') = \mathrm{B}_i(x')\chi + \rho_i$$

$$\tilde{\chi} = L[z - Hx',\, b_i^2 - \beta_i(x')],\ \tilde{x} = \tilde{x}' + \tilde{\chi}$$

Formal statements for different approaches to attitude determination by processing carrier phase measurements (page 3)

- Triple-differencing free of integer ambiguities and using special properties of conditionality

$$z'^{p}_{ij}(t_k) = z^{p}_{ij}(t_k) - z^{p}_{ij}(t_{k-1}) = \frac{(\rho_i(\overline{x}_j(t_k)) - \rho_z(\overline{x}_j(t_k))) - (\rho_i(\overline{x}_j(t_{k-1})) - \rho_z(\overline{x}_j(t_{k-1})))}{\lambda_{L_p}}$$

$$\overline{z}_1 = H_1\overline{x}_1 - H_0\overline{x}_0 = H_1(\overline{x}_1 - \overline{x}_0) - (H_1 - H_0)\overline{x}_0$$

$$\vdots$$

$$\overline{z}_k = H_k\overline{x}_k - H_{k-1}\overline{x}_{k-1} = H_k(\overline{x}_k - \overline{x}_{k-1}) - (H_k - H_{k-1})\overline{x}_{k-1} =$$

$$= H_k(\overline{x}_k - \overline{x}_0) - (H_k - H_0)\overline{x}_0 - (\sum_{l=2}^{k-1} \overline{z}_l - \overline{z}_1)$$

$$\left\| H_k - H_{k-1} \right\| \square\ 10^{-4} \square\ \sigma_{\Delta z}$$

Разности $\overline{x}_k - \overline{x}_{k-1} = \overline{v}_k$ *наблюдаемы в каждый момент времени*

Вектор $\overline{x}_0$ *наблюдаем при накоплении большого количества измерений*

Оценка разностей $\overline{x}_k - \overline{x}_{k-1}$ *и вектора* $\overline{x}_0$ *решает задачу на интервале времени*

Least-squares method (LSM) compared to LSM with restrictions

- Least-squares method is well formulated in variety of books

- LSM with restrictions is produced as iterative algorithm with linearized restrictions of equation type; least-squares solution is the initial value

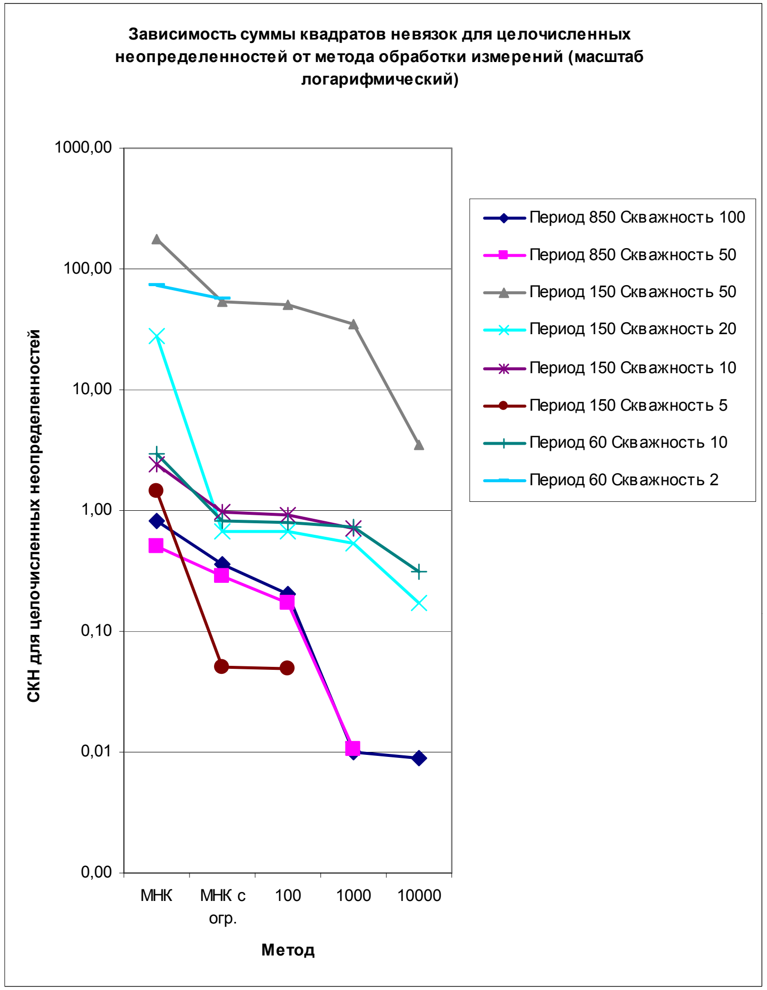
$$f_0(\xi) = \left(\zeta - \mathrm{H}\xi\right)^T \left(\zeta - \mathrm{H}\xi\right) \to \min_{\xi},\ f_i(\xi) = \xi^2_{n+3i+1} + \xi^2_{n+3i+2} + \xi^2_{n+3i+3} - b^2 = 0,\ i = 1..N$$

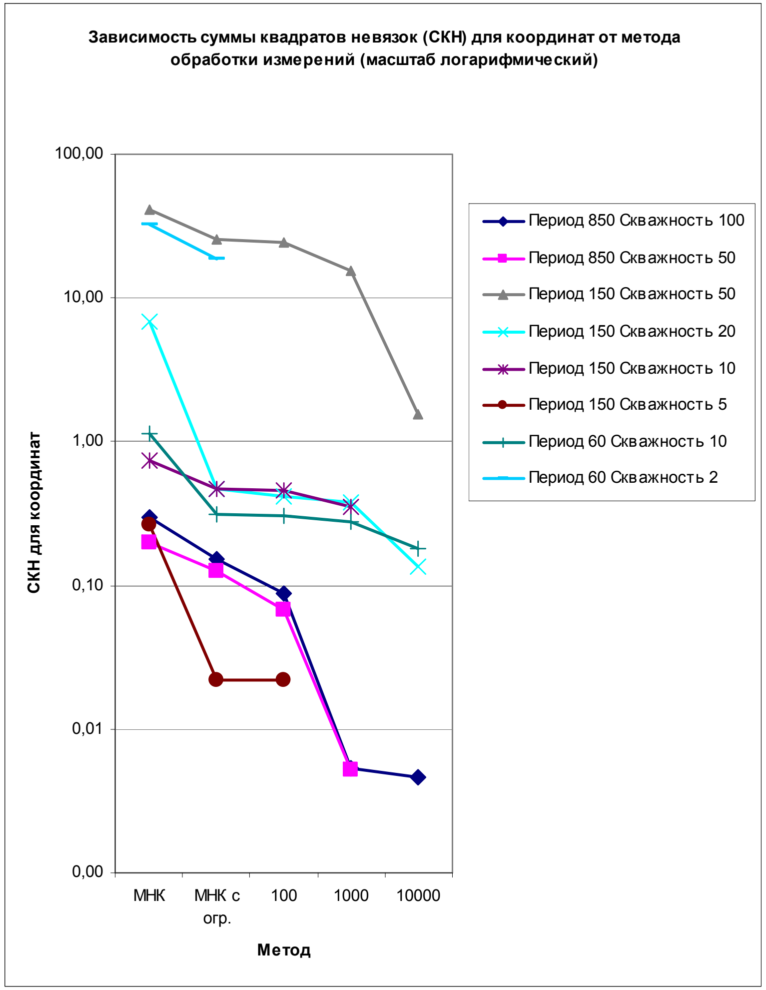
$$\varphi(\xi^{(k)}, \lambda^{(k)}) \square\ f_0(\xi^{(k)}) + \sum_{i=1}^{N} \lambda_i^{(k)} f_i(\xi^{(k)})$$

$$\begin{cases} \dfrac{\partial^2 \varphi(\xi^{(k)}, \lambda^{(k)})}{\partial \xi^2} h^{(k)} + \displaystyle\sum_{i=1}^{N} \delta_i^{(k)} \nabla f_i(\xi^{(k)}) + \dfrac{\partial \varphi(\xi^{(k)}, \lambda^{(k)})}{\partial \xi} = 0, \\ \left(\nabla f_i(\xi^{(k)}),\ h^{(k)}\right) + f_i(\xi^{(k)}) = 0. \end{cases}$$

$$\xi^{(k+1)} = \xi^{(k)} + h^{(k)},\ \lambda^{(k+1)} = \lambda^{(k)} + \delta^{(k)}$$

Least-squares method (LSM) compared to LSM with restrictions; actual measurements processing


Зависимость суммы квадратов невязок для целочисленных неопределенностей от метода обработки измерений (масштаб логарифмический)


Зависимость суммы квадратов невязок (СКН) для координат от метода обработки измерений (масштаб логарифмический)

Dynamics of observability during constellation evolution

- Let H_1 and H_2 be observation matrices for two epochs for m carrier phase measurements containing integer ambiguities

- Necessary and sufficient condition for observability is as follows:

$$\forall P_1 \in \square^{m\times m} \; rank(H_2 - H_1 P_1) = m$$

- The condition states that direction vectors to satellites are not transformed by a single linear operator; furthermore the difference from any that transformation would be enough to satisfy the condition

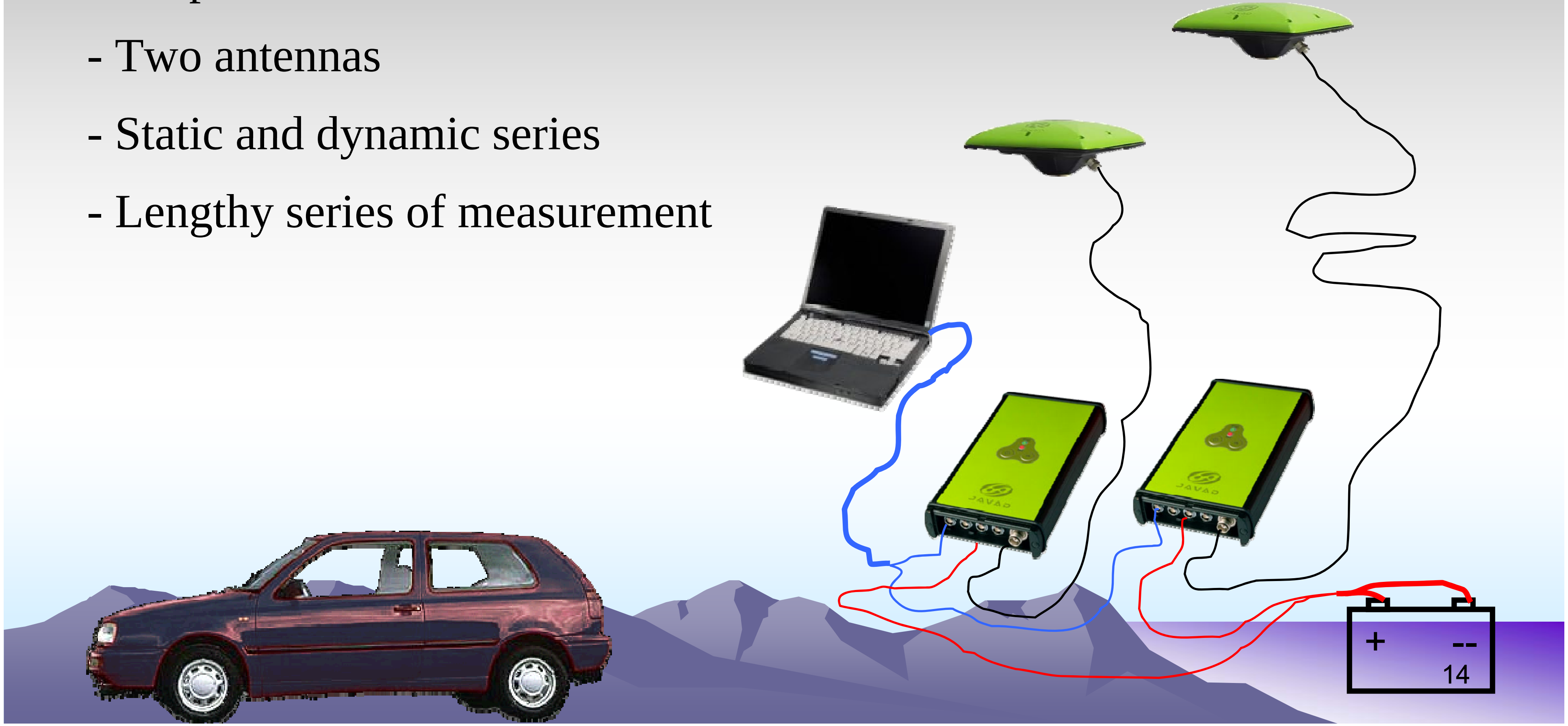
- The more suitable equivalent condition is the m-dimensional basis at the bottom $n - m$ rows of matrices:

$$U_1^T H_2 \text{ или } Q_1^T H_2, \text{ где } H_1 = U_1 S_1 V_1^T - \text{сингулярное}, \; H_1 = Q_1 R_1 - \; QR - \text{разложения соотв.}$$

$H_1 = U_1 S_1 V_1^T$ is singular value decomposition, $H_1 = Q_1 R_1$ is QR-decomposition

Field surveys

- Field surveys have been conducted several times in Moscow region and once near Tver
- “Topcon” hardware and software
- Two antennas
- Static and dynamic series
- Lengthy series of measurement



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