Recurrence Analysis of Solar-Terrestrial System

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Abstract

Recurrence is a characteristic of some dynamical systems like solar-terrestrial system. Modeling and predicting the behavior of this system will get more straightforward by revealing the structure of recurrences of states of the system, and by revealing the couplings between subsystems e.g. magnetosphere (or between other peripheral parameters like cosmic ray intensity). In this study, recurrence plots (RPs) are used to derive some characteristics of solar, geomagnetic and cosmic ray data. Indeed, recently developed methods of cross recurrence plot (CRP) and joint recurrence plot (JRP) are used to find the order of couplings between subsystems and peripheral data of space weather system. Both visual and quantitative analysis are presented here. For quantitative study, recurrence quantification analysis (RQA) is utilized and the results are employed to choose the best state space and best features to model or predict the system's behavior.

Keywords: Solar-Terrestrial System, Recurrence Plots, Recurrence Quantification analysis

1. Introduction

1.1. Solar-Terrestrial System

Sun as an active star causes many changes in interplanetary and space weather. For many years, sunspots are known to be signs of solar activity, but today we know they are not sufficient for modeling and predicting solar-terrestrial system. Scientists are trying to understand more about CMEs, Solar flares and solar wind. Solar wind as a medium for energy and particle transfer between Sun and Earth has typical speeds between 400 km/s and 750 km/s. High speed solar winds are originated from coronal holes, funnel-like regions of Sun's magnetic field [1, 2]. Effects of solar wind are observed in cosmic ray intensity decrease (Forboush effect), geomagnetic and ionospheric storms. Also, solar wind affects magnetosphere's shape.

Geomagnetic storms which we observe them as ring currents and large negative values for *Dst*, and *Kp* values more than 4, are subjects to damages and faults in human technology systems such as power lines (encountering with geomagnetically induced currents). Today, it is important to have more accurate models and predictors for solar-terrestrial system, and this will be possible with more knowledge about interrelations between variables of this system.

1.2. Recurrence, Cross Recurrence and Joint Recurrence Plot

More than 70 years after Poincare's pioneering work on recurrence property of dynamic systems, Eckmann et al., (1987) introduced recurrence plots for measuring time constancy of dynamical systems [3]. The main idea in recurrence plots is to determine whether two states of system are close in phase space or not. This leads to define recurrence matrix as below

$$R_{i,j}(\varepsilon) = \Theta(\varepsilon - \|\vec{x}_i - \vec{x}_j\|) \qquad (1)$$

Where $\vec{x}$ is state vector, ε is threshold distance and Θ is a discrete function defined as

$$\Theta(X) = \begin{cases} 1 & if\ X \geq 0 \\ 0 & othrwise \end{cases} \qquad (2)$$

$\|.\|$ is a metric (norm) which could be one of conventional norms such as Euclidean or L_∞-norm. Since symmetry is a property of any metric, the recurrence matrix is a symmetric matrix. This is obvious that the choice of ε affects our definition of recurrences of system, so it is important to make a reliable choice of that. One way for selection of ε is to consider it as 10% of mean distance between states.

The Recurrence Plot (RP) of system can be figured by means of the above recurrence matrix by plotting a black dot for 1's and nothing for 0's. Selection of ε will affect density of this plot. Another property of every RP is existence of the Line of Identity (LOI) which is due to reflective property of metrics ($\|\vec{x}_i - \vec{x}_i\| = 0 \overset{\forall i}{\Rightarrow} R_{i,i}(\varepsilon) = 1$). Some RPs encounter with false recurrence points (also called sojourn points), due to tangential motions. Tangential motion is due to closeness of states to their previous and next states, this means the temporal neighborhood leads to neighborhood in phase space. This effect is depicted in figure 1.

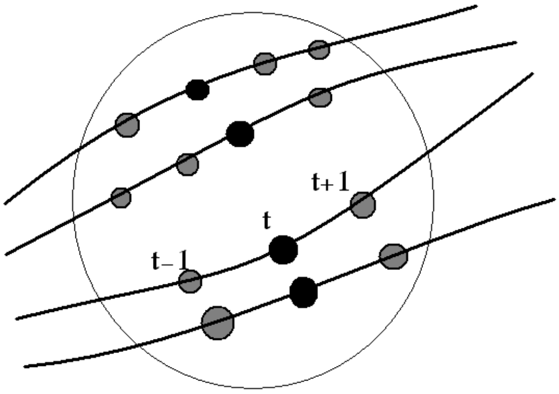
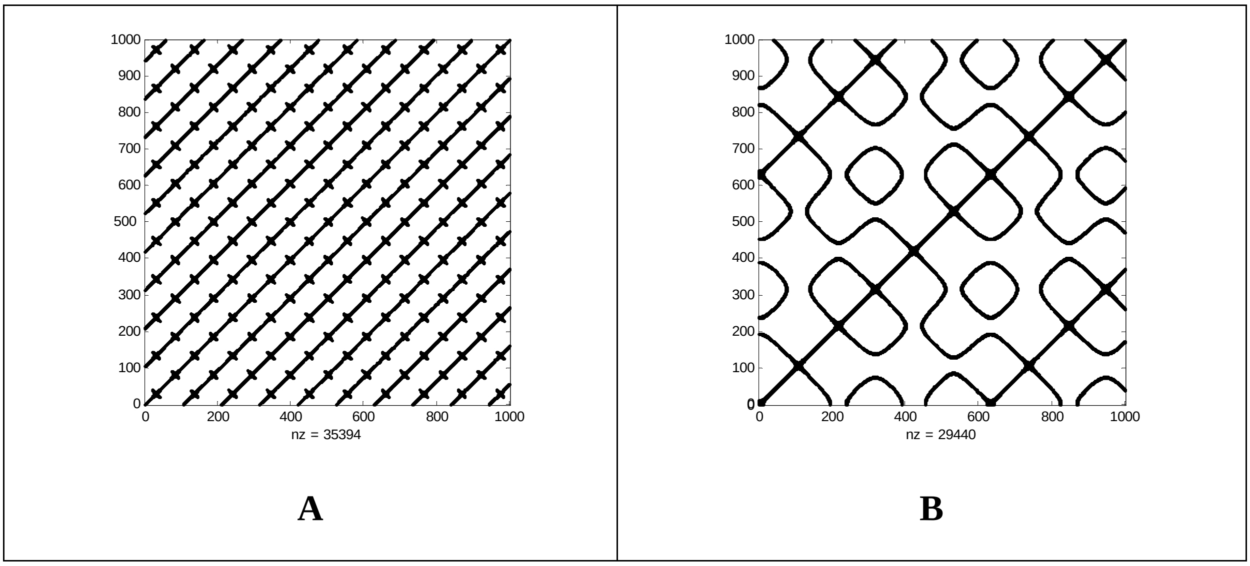
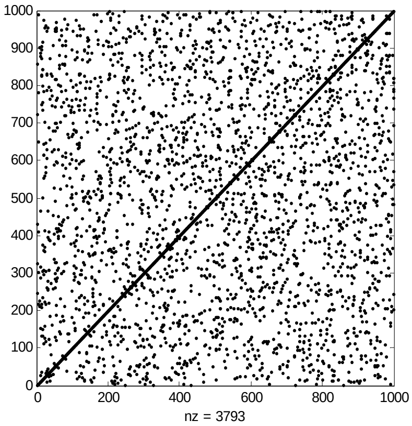


Figure 1. Grey points are sojourn points, due to tangential motion.

RPs show some characteristics of system through their shapes, for example periodic nature of system results in diagonal lines parallel to LOI, noisy nature makes recurrence points to be spread among entire RP. Marwan et al., explain the structures of recurrence plots [4, 5]. RPs for some systems are shown in figure 2.





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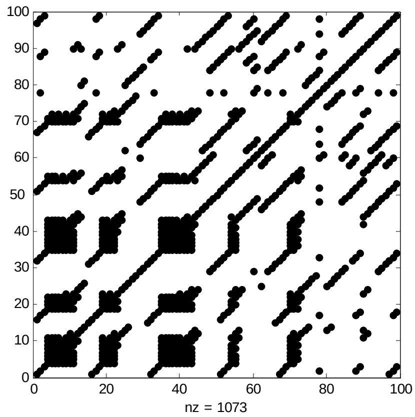

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Figure 2. (A) Periodic with 1 frequency (B) Periodic with 2 frequencies [$\sin(t) + 4\cos(3t)$].

(C) Random noise. (D) Chaotic system of logistic map [$x(t+1) = 3.679x(t)(1 - x(t))$]

Cross Recurrence Plot (CRP) is defined in same way as RP, except that, distance between two states from two different systems is considered here. So we define cross recurrence matrix as below

$$CR_{i,j}^{\vec{x},\vec{y}}(\varepsilon) = \Theta(\varepsilon - \|\vec{x}_i - \vec{y}_j\|) \quad (3)$$

It is important to normalize two systems to have a meaningful result. Then, CRP can be utilized to study relationships between two systems.

On the other hand Joint Recurrence Plot (JRP) is multivariable extension of RP. Joint recurrence matrix for two systems is defined as

$$JR_{i,j}^{\vec{x},\vec{y}}(\varepsilon^x, \varepsilon^y) = \Theta(\varepsilon^x - \|\vec{x}_i - \vec{x}_j\|) \times \Theta(\varepsilon^y - \|\vec{y}_i - \vec{y}_j\|) \quad (4)$$

This definition can easily get extended for more than two systems by multiplying their recurrence matrices in above equation.

$$JR_{i,j}^{\vec{x}(1,\ldots,n)}(\varepsilon^{x_1},\ldots,\varepsilon^{x_n}) = \prod_{k=1}^{n} R_{i,j}^{\vec{x}_k}(\varepsilon^{x_k}) \quad (5)$$

It is not necessary to normalize systems in the case of JRP, also the systems could have different dimensions.

1.3. Recurrence Quantification Analysis

For quantitative study of RPs, Recurrence Quantification Analysis (RQA) is developed in almost heuristic way, and it is an open area for further research. Some RQA measures are based on recurrence points, some are based on diagonal lines and some are based on vertical lines (or horizontal lines since for every vertical line there is a horizontal line in RP).
Recurrence Rate is defined as

$$RR(\varepsilon) = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j}(\varepsilon) \quad (6)$$

Where N is dimension of RP

This measure determines recurrence density of RP. It could be extended for case of CRP or JRP.

Another RQA measure, based on diagonal lines is determinism

$$DET = \frac{\sum_{l=lmin}^{N} lP(l)}{\sum_{l=1}^{N} lP(l)} \quad (7)$$

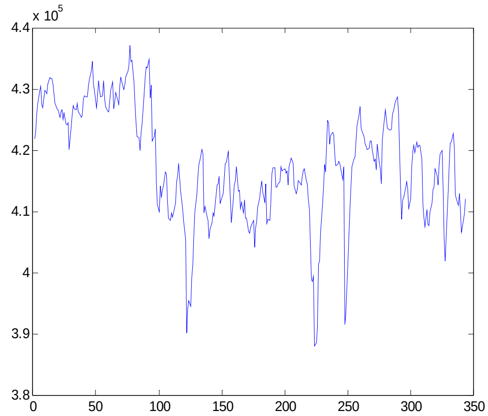
Where l denotes length of diagonal line, and $P(l)$ is number of diagonal lines with length l calculated by

$$P(l) = \sum_{i,j=1}^{N} \left(1 - R_{i-1.j-1}\right)\left(1 - R_{i+l,j+l}\right) \prod_{k=0}^{l-1} R_{i+k,j+k} \quad (8)$$

DET measures the predictability of system. For higher values of DET, system is more predictable. For some systems RR is high, but DET is low, this reveals a more noisy structure for that system.

2. Results

In this study we derive RPs, CRPs and JRPs for some indices of space weather. RPs for daily cosmic ray intensity from Climax station for year 1998 is depicted in figure .3(B). It could be seen that this data is encountered with extreme events because of white bands in RP, but other events are predictable because of regular structure for their vertical lines.



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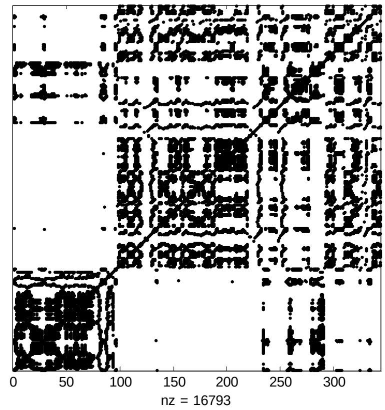

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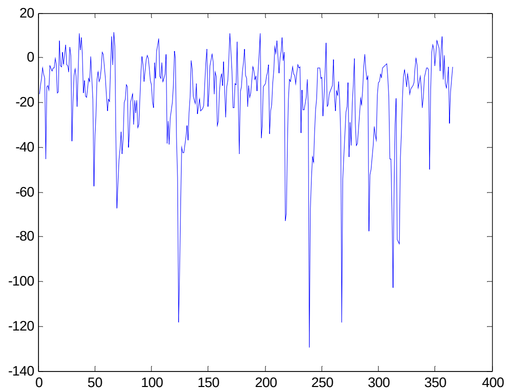
Figure 3. (A) Daily cosmic ray intensity of Climax station for year 1998 (B) RP for Climax data

RP for daily values of *Dst* for 1998 is shown in Fig. 4(B). It has not many big white bands, but its recurrences are in single dot patterns and this reveals low predictability.



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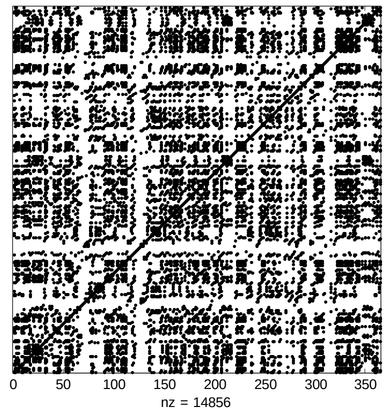

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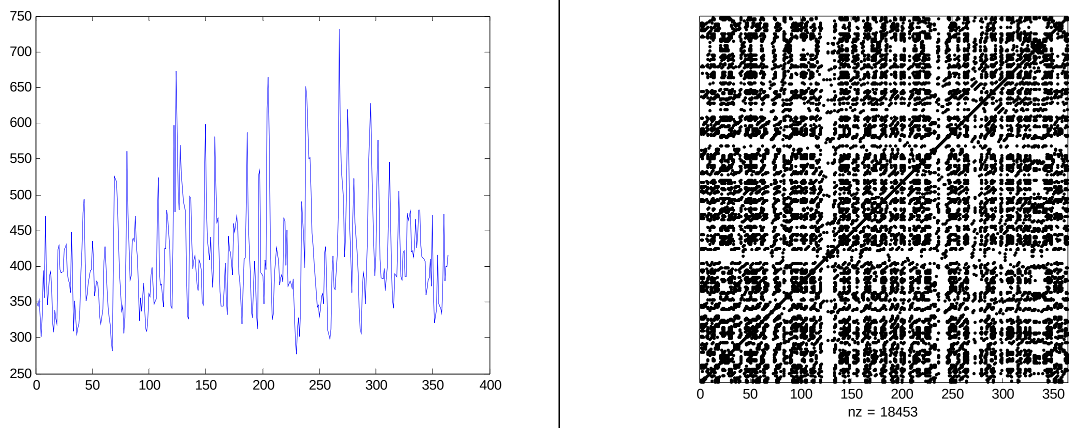
Figure 4. (A) Daily Dst index for year 1998 (B) RP for Dst data

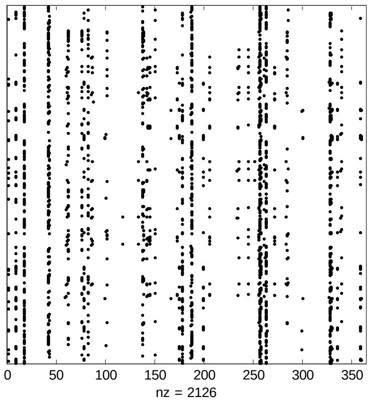
Same as for *Dst*, RP for solar wind speed of 1998 is depicted in Fig. 5(B). It is very similar to the case of *Dst*, but more recurrent structure.



A **B**

Figure 5. (A) Daily Solar wind speed for year 1998 (B) RP for solar wind speed data

CRP and JRP for *Dst* and solar wind speed is shown in Fig. 6. Because of different nature of these two variable with different dimensions (km/s and nT), CRP is not very easy to study, but JRP reveals same recurrence times and structures for these two indices.



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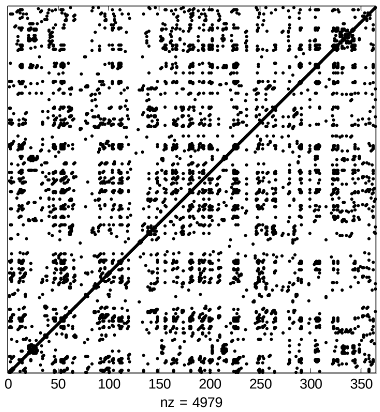

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Figure 6. (A) CRP of {Dst, Solar wind speed} for 1998 (B) JRP for {Dst, solar wind speed} data

Finally, RQA results of these three data are shown in table 1. It is easy to see the confirmation of RQA for visual study.

	RR	DET
Solar wind speed	0.2654	0.1060
Dst	0.1783	0.0808
Climax	0.1427	0.2379

Table 1. RQA results

3. Conclusion

Recurrence Analysis is useful for deriving characteristics of complex systems like Solar- Terrestrial System. In this study we found solar wind speed is more recurrent than *Dst* and CRI. Recurrences of *Dst* and Solar wind speed are almost similar, So Solar wind Speed might be an input to predictors of *Dst*. CRI has a more deterministic and predictable than *Dst* and Solar Wind by its past values. CRP is not as useful as JRP to study relations between space weather time series with different nature.

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