Experiments with particle bundles in cosmic rays: determination of strong-interaction parameters by methods of pattern-recognition theory

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A method is proposed in which simulated data and actual experimental data are analyzed jointly in a multidimensional space of measurable characteristics of extensive air showers. The identity and energy of the primary particle are determined through a solution of an inverse problem. Certain characteristics of the interaction of protons and nuclei with air nuclei are determined. In particular, the possibility of evaluating the inelastic cross section for the interaction of protons with air atoms and the inelasticity coefficients in PA interactions is discussed. These results, found through an analysis of model samples, raise the hope that it will be possible to formulate a cosmic-ray analog of a target experiment and to study the characteristics of PA and AA interactions at energies of 10^{15}–10^{17} eV.

1. INTRODUCTION

In cosmic-ray physics, the basic method used to solve ill-posed problems of determining primary physical characteristics (such as the mass composition and the energy spectrum of the primary cosmic rays and characteristics of the strong interaction) from the measured characteristics of extensive air showers (EASs) is to solve the direct problem, with a detailed simulation of the passage of the primary cosmic-ray particles through the atmosphere and through the experimental apparatus. The results of multidimensional simulations and experimental data are then compared.

In practice, one constructs an algorithm which describes the development and detection of various components of EASs at the observation level. This algorithm is based on some model for the process, i.e., on a set of parameters characterizing the flux of the primary cosmic rays and the interaction of hadrons and nuclei with nuclei of air atoms.

Numerical simulations with various models and comparisons of the results with actual experimental data make it possible to select a class of models which describe the experimental data satisfactorily. This approach makes it possible to reject a certain class of unsatisfactory models, but the available experimental data are not an adequate basis for selecting one unique model from the many which have been suggested. The difficulty is that we do not know the mass composition or energy spectrum of the primary cosmic rays, and we do not know the characteristics of the hadron–nucleus interaction at energies $E > 1000$ TeV. Our objective in the present study was thus to break up the problem of choosing a model into component steps. We first determine the identity of the primary nucleus, and then the energy; only then, after we have obtained "bundles" of protons and nuclei with a fixed energy, do we study the possibility of evaluating the parameters of the first interaction.

A classification method proposed in Refs. 1–3 makes it possible to determine the identity of the initial particle with an efficiency ~ 70–80% and to evaluate the energy of this particle within $\sim 25\%$ for each individual event. The results form the basis for implementation of the program outlined above.

In the present paper we are concerned with the third step of the program: estimating certain parameters of the hadron–nucleus interaction under the condition that the identity and energy of the initial particle are known. In particular, we discuss the possibility of evaluating the inelastic cross section for the interaction of protons with air atoms and the inelasticity coefficients in the PA interaction.

2. DETERMINATION OF THE IDENTITY OF THE PRIMARY PARTICLE AND ESTIMATE OF ITS ENERGY

The method of classifying EASs on the basis of multidimensional measurable characteristics is described in detail in Refs. 1 and 2. The parameters used in generating model events are given in the same places. The classification method is based on the Bayes decision rules with a nonparametric estimation of a multidimensional probability distribution. These rules take full account of the *a priori* information found as a result of the simulation.

The classification of events is carried out on the basis of the total number of electrons, N_e, and that of muons with $E_\mu > 5$ GeV (N_μ). To study the effect of the parameters of the strong interaction on the results of this classification, we examined the correlations between (on the one hand) the total number of electrons (N_e) and that of muons (N_μ) and (on the other) the total number of charged hadrons (N_{ch}), the average transverse momentum $\langle p_t \rangle_{ch}$, the total energy of the charged hadrons produced in the first strong interaction (ΣE_{ch}), the depth of the first interaction (z), and the primary energy E_0. Table I shows the corresponding correlation coefficients.

We see from this table that N_e and N_μ are essentially uncorrelated with N_{ch}, $\langle p_t \rangle_{ch}$, and z, while they depend strongly on the initial energy E_0 and on ΣE_{ch}.

Since N_e and N_μ are determined primarily by interactions of hadrons of the third and fourth generations, whose energy is comparable with the energy of modern accelerators, in which the characteristics of strong interactions have been studied thoroughly, by selecting events in a narrow energy interval (on the basis of N_e or N_μ, for example) we would expect that the results of a classification into different groups of nuclei would depend only slightly on the particular strong-interaction model selected for energies above ac-

TABLE I. Correlation coefficients expressing the correlation of N_e and N_μ with the parameters of the first interaction and the primary energy.

	N_{ch}	$\sum E_{ch}$	z	$\langle p_t\rangle_{ch}$	E_0
N_e	0,076	0,650	0,230	0,035	0,830
N_μ	0,270	0,830	−0,029	0,047	0,910

celerator energies. Here is a matrix of Bayes errors for $1\cdot10^5 < N_e < 2\cdot10^5$:

	P	CNO	H	VH
P	0,798	0,102	0,067	0,033
CNO	0,127	0,688	0,105	0,080
H	0,072	0,113	0,691	0,124
VH	0,034	0,090	0,150	0,726

The diagonal elements of this matrix show the probability for a correct classification of events corresponding to four groups of nuclei (P, CNO, H, and VH; protons and α particles are combined in one group). The nondiagonal elements show the probability for an erroneous classification. We see that the event classification efficiency is 70–80%, so that one can hope for a reliable selection of events initiated by a certain primary particle from the total number of showers detected.

In this study, the model showers were generated at zenith angles in the interval $0 \leqslant \theta \leqslant 45°$; a selection of events in a small interval of zenith angles may improve the classification efficiency.

The strong correlation of N_e and N_μ with the initial energy means that we can also work from these EAS characteristics to estimate the energy of the primary particle for events classified in one of the groups of nuclei.

We showed in Ref. 3 that the nonparametric regression method[4] makes it possible to estimate the energy of the primary particle within $\sim 25\%$. This method is based on the simple fact that events which are close together in a certain metric will have similar initial parameters (e.g., energies) in the space of measurable characteristics.

Model events were generated for primary protons and for primary nuclei with $A \geqslant 24$. The initial energy was generated in a random fashion, uniformly over the interval $100 \leqslant E_0 \leqslant 10^4$ TeV.

An analog of an experimental sample was found by sampling the model events by the procedure of Refs. 1 and 2, from model showers generated in accordance with the model of quark–gluon strings[5] under the assumption of a mixed composition and a power-law energy distribution of the primary cosmic rays, as in Ref. 6.

Here are the relative mean-square errors in the estimates of the energy of the "pseudoexperimental" events initiated by protons and by nuclei with $A > 24$, along with the sizes of the model and control samples from which the energies were estimated:

$\langle N_e\rangle/10^5$	0,66±0,196	1,404±0,281	2,716±0,534	9,758±1,079
$\langle N_\mu\rangle/10^3$	2,74±0,258	3,839±0,335	5,881±0,667	14,545±1,01
N_{0B}^{P}	864	913	484	402
$N_{\exp}^{P}$	439	465	256	216
CKO^P, %	20	24,3	25	25
N_{0B}^{A}	377	357	184	128
$N_{\exp}^{A}$	184	225	102	73
CKO^A, %	10,1	10,6	9,7	

The nonparametric-regression method is actually a generalization of the classification method to the case of an infinite number of classes, differing in continuously varying parameters (energy, interaction cross section, average multiplicity, etc.).

By using methods based on pattern-recognition theory, one can thus accurately determine the identity and energy of the primary particle, without leaning on a model of the strong interaction. It therefore becomes possible to take up a study of the characteristics of hadron–nucleus interactions at energies above those attainable in accelerators, by studying various components of EASs.

3. POSSIBILITY OF DETERMINING CHARACTERISTICS OF THE STRONG INTERACTION FROM THE DATA FROM COMPLEX INSTALLATIONS

In an effort to study the possibility of determining the parameters of the strong interaction of protons with air nuclei from data from complex installations (under the condi-

TABLE II. Correlation coefficients showing the correlation of the parameters of the first interaction with measurable quantities.

	N_{ch}	ΣE_{ch}	ΣE_{ch-p}	$\Sigma E_{ch\pi}$	z	$\langle p_t\rangle_{ch}$
$\langle R_\mu\rangle$	0,167	−0,145	−0,087	−0,084	−0,615	0,056
σ_R^{μ}	0.150	−0,100	−0,052	−0,049	−0,590	0,012
$\langle E_\mu\rangle$	0,035	−0,044	−0,011	−0,006	−0,210	0,042
σ_E^{μ}	0.030	0,040	0,053	0,057	−0,120	0,032
N_e	0,076	0,652	0,549	0,537	0,230	0,035
N_μ	0.270	0,831	0,735	0,713	−0,029	0,047

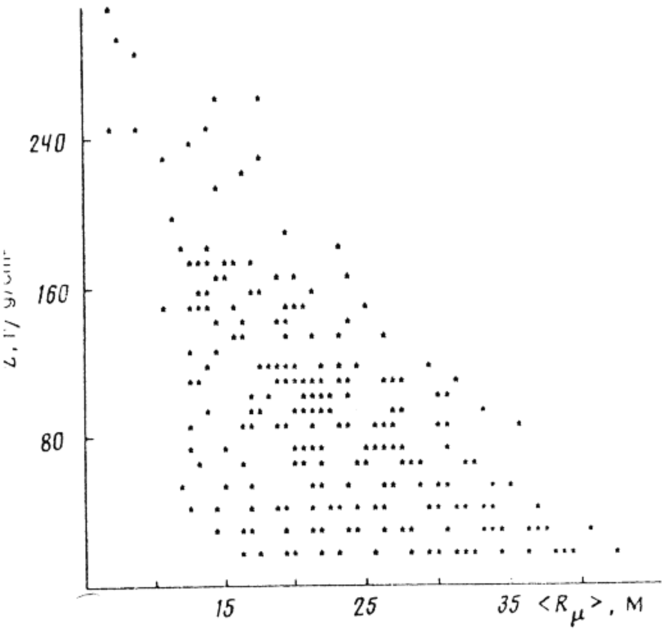

FIG. 1. Average distance from the shower axis for muons with $E > 200$ GeV as a function of the mean free path.

tion that the identity and energy of the primary particle have been determined), we generated some model showers initiated by primary protons with an energy $E_0 = 1000$ TeV. We calculated the correlation coefficients for the correlation of various measured characteristics of a shower with the parameters of the first interaction.

We considered the following parameters of the first interaction: z, the depth of the first interaction; N_{ch}, the number of charged hadrons; ΣE_{ch}, the total energy of the charged hadrons; ΣE_{ch-p}, the total energy of the charged hadrons other than protons; $\Sigma E_{ch\pi}$, the energy of the charged pions; and $\langle p_t \rangle_{ch}$, the average transverse momentum of the charged hadrons.

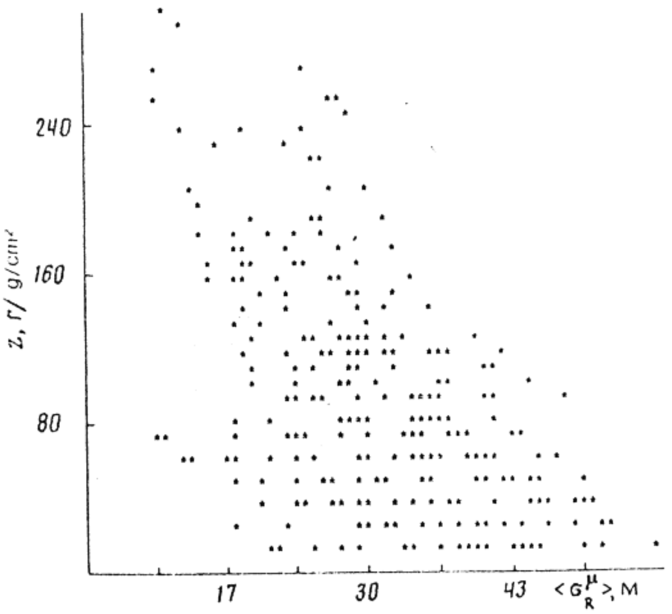

FIG. 2. Standard deviation of the spatial distribution of muons with $E > 200$ GeV versus the mean free path.

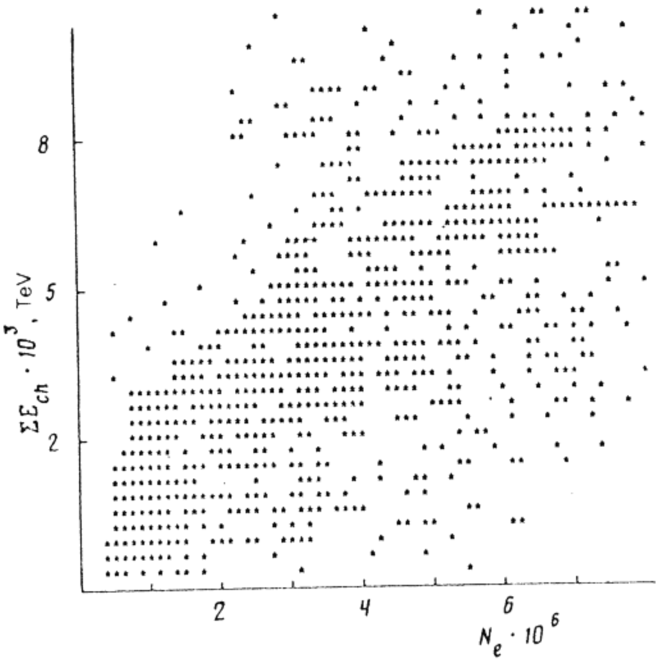

FIG. 3. Total number of electrons versus the total energy of the charged hadrons produced in the first interaction.

Table II shows correlation coefficients for the correlation between these parameters of the first interaction with N_e, N_μ, and the moments of the spatial and energy distributions of muons with $E_\mu \geqslant 200$ GeV (the average distance from the shower axis, $\langle R_\mu \rangle$; the average muon energy $\langle E_\mu \rangle$; and the standard deviations of the spatial and energy distributions, $\sigma_{R,E}^\mu$). It follows from this table that N_e and N_μ are correlated fairly strongly with the energy characteristics of the first interaction (ΣE_{ch}, ΣE_{ch-p}, $\Sigma E_{ch\pi}$), while the moments of the spatial distribution of the high-energy muons depend on the depth of the first interaction, z.

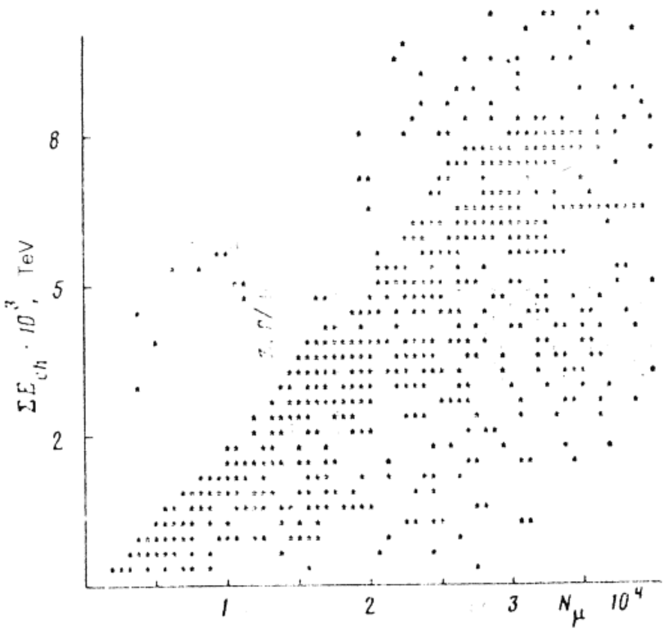

FIG. 4. Total number of muons with $E > 5$ GeV versus the total energy of the charged hadrons produced in the first interaction.

TABLE III. Actual and reconstructed values of the mean free path and the inelasticity coefficients.

	Actual value	Estimate		Actual value	Estimate
$\langle \Sigma E_{ch} \rangle$	612±5	628,3±15,9	$\langle \Sigma E_{ch}(\pi) \rangle$	361±3	368±12,3
$\langle \Sigma E_{ch}(\pi, k) \rangle$	411±9	421±12	$\langle z \rangle$	55,4±2,4	52,7±3,2

These conclusions are supported by Figs. 1–4, which show scatter diagrams of the parameters of the first interaction and of the measurable characteristics of EASs.

We can thus work from N_e and N_μ to estimate the inelasticity coefficients. We can work from the shape of the spatial distribution of high-energy muons to estimate the depth of the first interaction and, correspondingly, the cross section for the inelastic interaction of protons with air nuclei. The statistical independence of these EAS characteristics and of other parameters of the first strong interaction (Table II) indicates that the estimates of the corresponding quantities will be somewhat model-independent.

In this study we used a nonparametric-regression method to estimate the inelasticity coefficients and the inelastic cross section for the interaction of protons with air nuclei at an energy $E_0 = 1000$ TeV. In this method, each individual event (a point in the multidimensional space of measurable characteristics) is a subject of analysis.

Since the interactions of hadrons and nuclei and the development of the nucleus–electron cascade in the atmosphere have a probabilistic nature, there is a certain probability that each individual event will correspond to any model. The task of the analysis is thus to select the most probable model for each event on the basis of the measurable characteristics.

In the generation of model showers at particle energies above 100 TeV, the parameters of the strong-interaction model such as the average multiplicity, the cross section, the average transverse momentum, and the inelasticity coefficient were generated in a random manner, uniformly over a wide range. At energies below 100 TeV, the simulation was carried out in accordance with the model of quark–gluon strings. (As the average multiplicity varies at a fixed energy, the inclusive spectrum also changes.) In this manner we obtained a "bank" of possible models. The randomly generated events correspond to different models, and all the events in the model bank have the same probability.

To test this method, we then took up the problem of studying the interaction of a "bundle" of primary protons with atmospheric nuclei. The proton energy was generated in a random fashion in accordance with a normal distribution with mean energy 1000 TeV and a standard deviation of 20%. The EASs were then generated in a random fashion with the help of the characteristics of one specific model from the bank of possible models.

For each pseudoexperimental event **x**, we determined its nearest neighbors from the bank of possible models (the number of such neighbors ranged from 3 to 7) in the space of the best measurable EAS characteristics, which were selected beforehand. (These characteristics will of course differ from one specific problem to the next.) The corresponding characteristics of the first interaction were then estimated from

$$\hat{E}(\mathbf{x}) = \sum_{i=1}^{K} C_i E_{(i)}(x), \quad \sum C_i = 1,$$

where K is the number of nearest neighbors, and $E_{(i)}$ is the value of the characteristic of the first interaction (the mean free path, the inelasticity coefficient, etc.) of that model event which is the ith nearest neighbor of the experimental event **x**. The weight factors C_i are inversely proportional to the distance.

This formulation of the method makes it possible to evaluate the accuracy of the method, since we temporarily forget about the characteristics embodied in the particular model that is selected. Later, after these characteristics are reconstructed, we calculate the distribution of relative errors.

The results of these numerical simulations are shown in Table III. Also shown there are the particular values [illegible] the corresponding parameters of the strong interaction of protons with air nuclei which we used in generating the pseudoexperimental showers. It can be seen from this table that the actual and reconstructed values of the mean free path and the inelasticity coefficients agree well.

Figure 5 compares the actual and reconstructed integral distributions of the depth of the first interaction. We see that these distributions are the same, within the statistical errors, although there is some steepening of the reconstructed distribution. Figure 6 shows the distribution of the relative error of the reconstructed inelasticity coefficient:

$$D = \left\{ \sum E_{ch}^{est} - \sum E_{ch}^{tr} \right\} / \sum E_{ch}^{tr}.$$

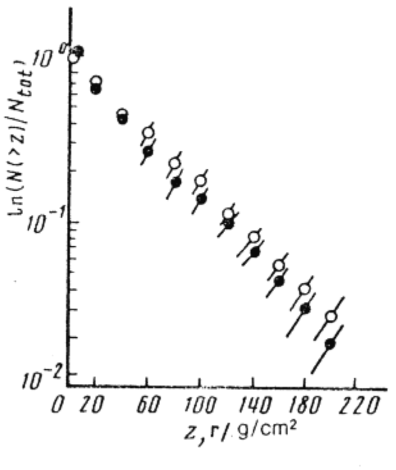

FIG. 5. Distribution of the relative error in the estimate of the inelasticity coefficient in the PA interaction at $E = 1000$ TeV.

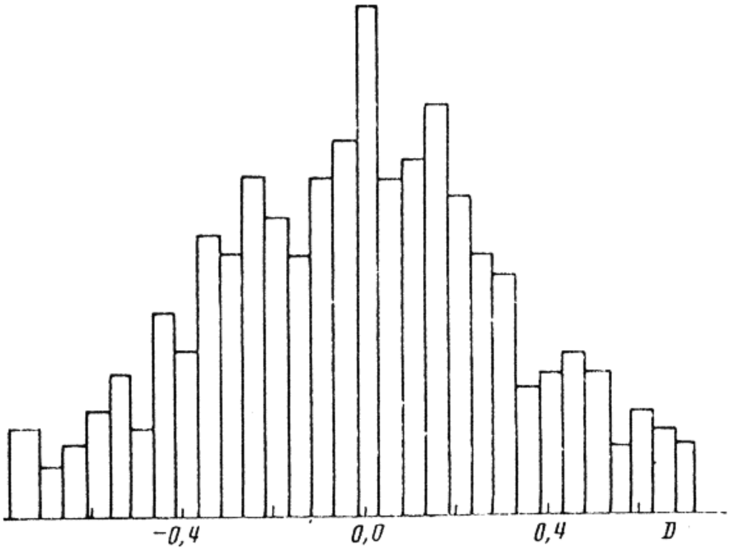

FIG. 6. Frequency histogram of the relative error in the estimate of the inelasticity coefficient.

It can be seen from this figure that the estimate of the inelasticity coefficient is unbiased and that the error in the determination of K_{ch} in each individual event is on the order of 35%.

The results of these numerical simulations thus show that one can work from the characteristics of the electron–photon and muon components of EASs to determine very accurately the inelastic cross section and the inelasticity coefficients in proton–nucleus interactions at energies above those attainable in accelerators.

We repeat that the results found here are not based on some specific model of the strong interaction with fixed parameters (at energies above the accelerator range).

4. CONCLUSION

This new method of jointly analyzing simulated and experimental data, which is itself based on methods developed in theory of pattern recognition, makes it possible to select a single model from the bank of possible models. Specifically, one can do the following:

a) One can determine, with an efficiency of 70–80%, the identity of the initial particle. One can estimate its energy within an error of ~25%. The results of the classification are essentially independent of the parameters of the hadron–nucleus interactions at energies above the accelerator range.

b) One can find model-independent estimates of certain parameters of the strong interaction at energies above the accelerator range by working from measurable characteristics of the electron–photon and muon components of EASs.

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