NEW ALGORITHMS FOR γ-QUANTA ENERGY ESTIMATION BY VHE γ-RAY TELESCOPES WITH THE CHERENKOV LIGHT IMAGING FACILITIES.

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Abstract

The successful application of the multidimensional analysis technique for discrimination of γ-quanta Cherenkov patterns from background hadrons observed from the Crab Nebula by means of the Whipple observatory telescope that permits an improvement of the signal-to-noise ratio by up to ~50%, makes it possible to pose the problem of estimation of the energy of an individual γ-quantum.

The mean-square error obtained in this work is to be 25-30%, giving us hope of a reliable estimation of the energy of the γ-quanta from the Crab Nebula registered at the Whipple observatory.

Introduction. The first work on numerical estimation of the energy of individual γ-quanta seems to be [1], where the polynomial form of the two parameters, the total Cherenkov light and the flash pattern centre coordinates, is taken as an estimate. The polynomial form coefficients were sought for in two energy ranges separately. The mean-square error turned out to be within 30-40%.

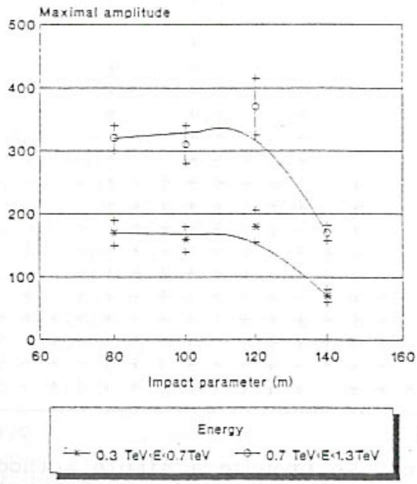
In the present work it is shown that in the case of dividing all the simulation data into events fallen into separate zones - concentric circles consisting of separate photomultipliers - one can ignore the impact parameter, i.e. dependence on the Cherenkov spot centre coordinates is removed. Besides, we did not use the pre-selected form of functional dependence of energy on the flash parameters measured. The simulation data are used in the estimation procedure. This, in our opinion, permits one to avoid additional errors due to approximation of the events having undergone strong random fluctuations.

The Nonparametric Regression Method. Suppose, a flux of particles is sporadically and independently incident on the atmosphere in accordance with some spectrum f(E). Then these particles, undergoing random collisions and interactions with air nuclei, initiate an extensive air shower, the parameters of which are registered by the experimental setup, i.e. each value of E is put into coincidence with some random vector of measurements, $\vec{X}$, according to some conditional probability density $P(\vec{X}/E)$.

The peculiarity of the solution of the regression problem in the cosmic-ray physics is the fact that neither method parameters k = 3,5,7,9,11. The weights were taken to be inversely proportional to the distance from the estimated event to its nearest neighbor.

<u>The γ-quanta energy estimation.</u> Hillas and Patterson have mentioned that the radial distribution of the Cherenkov light from the EAS initiated by gamma-rays (with respect to the centre of the field of view of the telescope) is quite stable [4], which is confirmed by our calculations, performed using simulations of new Whipple telescope[5] (see fig.1).

Fig.1 Maximal tube amplitude dependence
for Whipple observatory telescope.

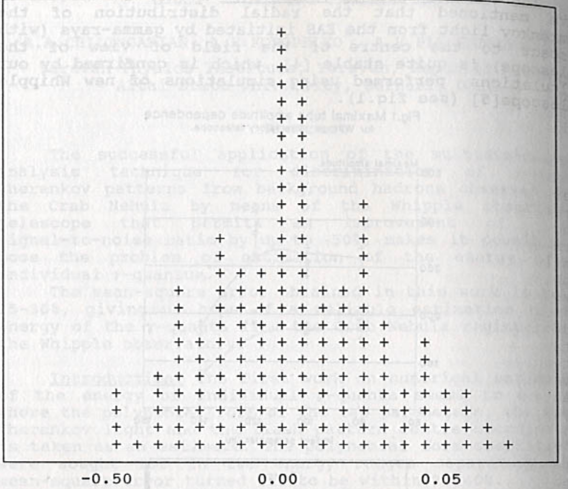


As it is seen from the correlation matrix calculated over the events fallen into zone 3 , the correlation of the two highest intensities AMP1 and AMP2 with Impact parameter are negligible. On the other hand, very strong correlation of these parameters with initial energy E_γ indicates linear dependence of the quantity of light on the initial energy, thus giving a reason for estimating the energy only by intensity of the light.

The correlation matrix of γ- image parameters

	E	IMPACT	AMP_1	AMP_2
E	*			
IMPACT	-0.119	*		
AMP_1	0.927	-0.222	*	
AMP_2	0.925	-0.252	0.952	*

The estimation was carried out in the space of two maximal tube intensities AMP1, AMP2, the number of the nearest neighbors used was 7, the obtained half width of distribution of relative errors of estimation was 0.25.



<u>Conclusion.</u> We propose a simple method of estimation of the energy of gamma-rays registered by Cherenkov telescopes. The method employs the very strong correlation of intensity of light with the initial energy in case of separate zone-by-zone event analysis. The high estimation accuracy (25-30%) together with the technique of multidimensional γ-ray event selection allows us not only to investigate the spectra of discrete sources in the energy range of 0.1 to 10 TeV, but also to carry out investigations of the interaction of very high energy "γ-beams" with the atmospheric target.

References

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the true spectrum f(E) nor the conditional density $P(\vec{X}/E)$ are known in the general case, but there is a training sequence $\{E_i, X_i\}$, $i=1,M_{TS}$ (obtained by simulation) and it is required to "recover" the regression $E=E(X)$ by this sequence (M_{TS} is the number of events in the training sample).

In the absence of systematic errors $M\{X\}_E=X(E)$ (the mathematical expectation of random vector measurement at a fixed independent variable (energy) is equal to the regression function value in that point) this problem is reduced to one of minimization of the average risk

$$I(\alpha) = \int (E-F(\vec{X},\alpha))^2 P(X,E)\, dX\, dE, \qquad (1)$$

where $F(X,\alpha)$ is some functional family depending on the parameter α, $P(X,E) \sim P(X)P(X/E)$ is the probability density function. If a priori information is available about the form of the probability function and the chosen functional family $F(\vec{X},\alpha)$ is not too complex, then the regression problem can be solved by the least mean squares or the maximum likelihood standard methods.

Due to the complicated stochastic picture of particles and nuclei passing through the atmosphere and the detectors, we do not expect a standard probability interpretation of all random processes, which is why we have chosen a method based on a nonparametric way of treatment of a priori information, which does not impose any structure and totally uses the information carried by TS.

The method is based on the obvious fact that the events close to some metric (usually the Mahalanobis metric is used) in the feature space have similar energy - the compactness hypothesis. The method based on consideration of the "nearest neighbors" is first analyzed in Ref.[2]. In this work it was shown that when the number of the nearest neighbors, K, and the total number of events in TS, M, tend to infinity so that $K/M \to 0$, then the risk of the procedure tends to the minimum achievable Bayes risk and even the use of one neighbor increases the risk only twice as compared to the Bayes risk. The uniform consistency of the following estimate is shown in Ref.[3]:

$$\hat{E}(X) = \sum_{i=1}^{k} C_i E_{[i]}(X), \quad \sum_{i=1}^{k} C_i = 1, \qquad (2)$$

where $E_{[i]}(X)$ is the value of the independent variable (energy) of the i-th nearest neighbor of the event $\vec{X}$ in the feature space.

Despite the fact that the nonparametric procedures are optimal under unlimited sampling, for the case of finite samples there are practically no theoretical and practical recommendations on the choice of the method parameters (e.g., the number of nearest neighbors).

For best parameter value selection, several estimates were simultaneously calculated corresponding to different