Some details on the accuracy of energy estimation

Answers to criticism of Dr. Sokhoyan on the reliability of the results of *"Light and Heavy Cosmic Ray Mass Group Energy Spectra as Measured by the MAKET-ANI Detector"*

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The current report aims to answer some questions and criticism on the reliability and the quality of results of "Light and Heavy Cosmic Ray Mass Group Energy Spectra as Measured by the MAKET-ANI Detector" [1].

The problem is the estimation of the primary energy of the light and heavy groups of primary cosmic rays. The EAS events registered by MAKET-ANI installation and full simulations of EAS including the MAKET-ANI response using CORSIKA program with QGS strong interaction model are the basis of performed analysis. Taking into account the detector response function is an essential point when analyzing experimental data and making physical inference.

The details of the data analysis technique (two way classification and energy estimation of EAS measured by the MAKET-ANI detector) and obtained results can be found in [2], in this note we demonstrate only the reliability and the quality of the performed analysis and obtained results as published in the referred journal.

Dr. S. Sokhoyan in his quarterly report of the 4th quarter of A757 project [3] has performed an energy estimation of the light and heavy group of nuclei (based on the light-heavy EAS classification results obtained in [2]) and compared the obtained results with the reported results in [2]. In [3] author has performed the energy estimation of each group of nuclei in 5 different zenith angle bins separately using log-normal fit of $N_e - E_0$ dependencies. While in [2] we estimate the energy of each group of nuclei for all zenith angles: $0 - 45°$, using Neural Network estimator. The energy estimation method in [3] is based on the coefficients of transition from measured EAS size to the primary energy. These parameters are obtained by means of simulated data, fitting the MC $N_e - E_0$ dependence by a straight line.

In his report the author concludes:

From Fig.6 we deduce that an agreement of (NN) - method with CORSIKA for any N_e *is observed only for vertical events. With increasing* θ *one recognizes a systematic rising of energy values obtained by (NN) - method in ranges* $5 \leq Log(N_e) < 5.7$*, and* $Log(N_e) > 6.6$*. This may result in differences in the shapes of* E_0 *- spectra for different zenith angles... In case of "heavy" nuclei, the good agreement of (NN) with CORSIKA is observed only in the first and fourth* θ *interval and for all angles* $\theta < 39°$ *and agreement only at* $6.2 \leq Log(N_e) < 7$ *is observed.* (see figures 1 and 2, which shows the primary energy dependence on EAS size).

In plots presented by Dr. Sokhoyan the results obtained by his method are marked as **by this work** - meaning the $N_e - E_0$ linear fit, and the results obtained by our method as **by number four** - meaning the Neural Network estimation.

Here we discuss these figures and point out what is incorrect in comparisons and conclusions presented by Dr. Sokhoyan.

- As one can see in figures(1, 2) of [3], by the solid line is shown the $E_0 - N_e$ dependence taken from CORSIKA simulations. And this is the first incorrectness of the statistical model used. Although the EAS size is highly correlated with the primary energy and the correlation coefficient reaches ~ 0.95 for both, light and heavy group of nuclei, this still does not mean that the $N_e - E_0$ dependence is a straight line. Hence, first of all, Dr. Sokhoyan has used not adequate model to fit this dependence by a line and use fit coefficients for the primary energy estimation, and second, one has to compare the experimental points with the MC data points, but not with the fitted line.

- furthermore, it is not correct to compare the experimental results which include definite methodical errors (limited accuracy of energy estimation and mass classification) with MC light and heavy events which do not contaminate misclassified events from alternative classes. One has to classify and estimate the MC data in the same way as experimental ones. Using MC control samples we have estimated the mean misclassification to be $\approx 25\%$, and the energy resolution $\approx 30\%$ for the light and $\approx 20\%$ for heavy group of nuclei respectively. (one should not forget also the presence of intermediate nuclei in both light and heavy groups of nuclei in experimental points, and essential point when making conclusion on the quality of agreement between experimental and MC points).

- it is difficult to understand what tell us the statistical error bars used on these plots, where the primary energy versus the EAS size are plotted. When making comparisons of the mean values of parameter distributions in bins, and making conclusion on the closeness of different points, one should specify also the deviation of the distributions in that bins. So, instead of statistical errors, one should use in such plots the spread option, which shows the RMS error of the E_0 distributions (energy resolution) in different N_e bins, like it is done in figures 3, 4, 5, 6, 7, 8. From these figures one can see that within the errors, simulated and experimental points are in well agreement. Some negligible discrepancy is observed between experimental and MC points for the light group of nuclei in first θ bin and for the heavy group of nuclei in last θ bin.

Thus, the figures presented by the author of [3] can not be used to make a conclusion about distortions of the energy spectra.

For comparison of energy estimation quality by different methods one should use much more informative criteria. That is the dependence of the relative error of estimation on the primary energy, for all particles in different zenith angle bins (see figure 15). And we will be happy to compare our estimator with the linear estimator when Dr. Sokhoyan presents the results in a proper form.

Taking into account the above mentioned points we have performed the same kind of comparisons, which are presented in figures 9, 10, 11, 12, 13 and 14. From these figures, which show the EAS size dependence on the estimated primary energy for light and heavy nuclei, one can see the very good agreement between estimated experimental and MC energies for all slant depths in the primary energy range where we demonstrate an unbiased and accurate energy estimation, i.e. $10^{15} - 2 \times 10^{16}$ eV. Some slight underestimation of energy for the heavy group of nuclei is observed in the lowest energy bins for the last two zenith angle bins, such that an unbiased estimation starts at 6.1 and 6.2 in lgE_0.

From this figure it is easy to see that the energy estimation is very accurate and almost unbiased in the $0 < \theta < 39°$ zenith angle range. The bias of estimation over the energy range $6 \leq lgE_0 < 7.5$ does not exceed the 5% level and the estimation accuracy increases with increasing energy.

For the first zenith angle bin one observes slight overestimation of energy in some bins, which does not exceed 10%. For the second, third and fourth θ bins one can see accurate and unbiased estimation of energy, except for the $lgE_0 = 6$, where some $\sim 10 - 20\%$ overestimation occurs. Only in last fifth θ bin one observes $\sim 10\%$ underestimation for high energies and $\sim 30 - 40\%$ overestimation at lowest energies.

Comparing these plots with ones in figure 16, 17 where the same dependencies are shown for whole θ range but for light and heavy primaries separately, one can conclude that the accuracy of estimation is significantly higher for heavy primaries, but some overestimation at low energies and underestimation at intermediate energies (both not exceeding 5%) are observed. For the light group of primaries energy resolution is not as good as for heavy primaries, but the estimation is unbiased over whole energy range.

Further, the author of [3] writes:

...But in results of (NN) - method (see Fig.8(b)) the shift of E_0 - spectral intensities with θ at $Log(E_0/GeV) \approx 6$ almost 3 times is observed. With increasing E_0 the shift is decreased to ≈ 1.5 times at $Log(E_0/GeV) = 7$, and the energy spectrum for "light" primaries and, as consequence "All Particles" spectrum, constructed for $\theta \leq 30°$ are unreliable. A better agreement between two methods is observed in the E_0 - spectra constructed in case of the (Si+Fe) group: in the region $6.3 < Log(E_0/GeV) < 7$ are no distortions in shape of (NN) energy spectra on first 4 depths. But one cannot analyze the energy spectrum for existence of absence of the knee in the such a narrow energy interval...

and the presented figures are 18 and 19.

These figures, which compare the primary energy spectra for light and heavy primaries for different zenith angle bins obtained by the author and by our method look very strange. Here we simply present the Figure 20 which demonstrates the same spectra obtained and constructed by us. Just the first look on these figures brings only to a question how the spectra were constructed by the author of [3] using the experimental data classified and estimated by our NN method (provided to him for comparative analyses).

So, we think it is not necessary to discuss the problems pointed out by the author of [3], since the spectra he has constructed are completely different (simply wrong) from what we have obtained by our analyses in fact, and the mentioned problems disappear in the view of true plots and results which we present in this paper.

It is worth to mention that estimating the primary energy in the wide zenith angle range $(0-45^{\circ})$ and then observing very good agreement of the spectra constructed for 4 zenith angle bins, for both, light and heavy groups of nuclei (see figure 20), demonstrates (proves) the correct and accurate energy estimation by our method described in [2].

Some discrepancy of the spectra constructed for the fifth zenith angle bin is observed, which could be expected taking into account results from figures 15 and 13, but the statistics in this last θ bin is $\sim 4\%$ for the light and $\sim 3\%$ for the heavy groups of nuclei from the overall experimental events analyzed. So, the shape and parameters of the spectra over the whole θ range for both, light and heavy groups of nuclei can not be significantly disturbed.

References

[1] Chilingaryan A. A., et al.: Light and Heavy Cosmic Ray Mass Group Energy Spectra as Measured by the MAKET-ANI Detector. *The Astrophysical Journal*, **603**; L29-L32, 2004

[2] Vardanyan A. A.: EAS data classification into light and heavy mass groups by MAKET installation. Report on 3rd quarter of grant A757, April 2003

[3] Sokhoyan S. H.: Comparing analyses of E_0 determination methods on the MAKET ANI installation and primary energy spectra for (H+He) and (Si+Fe) nuclei groups in range of $3 \times 10^{14} - 10^{17}$ eV.

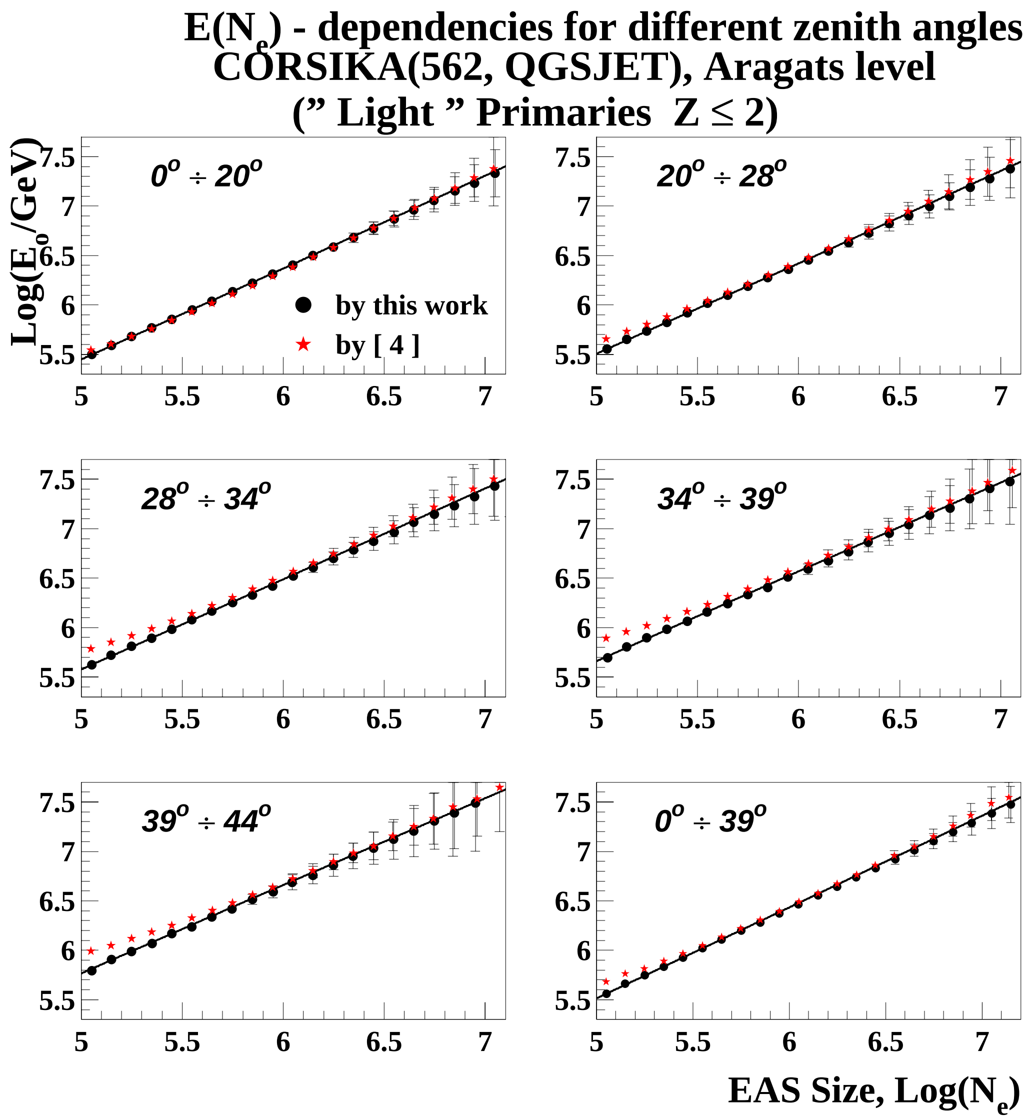


Figure 6:

Figure 1:

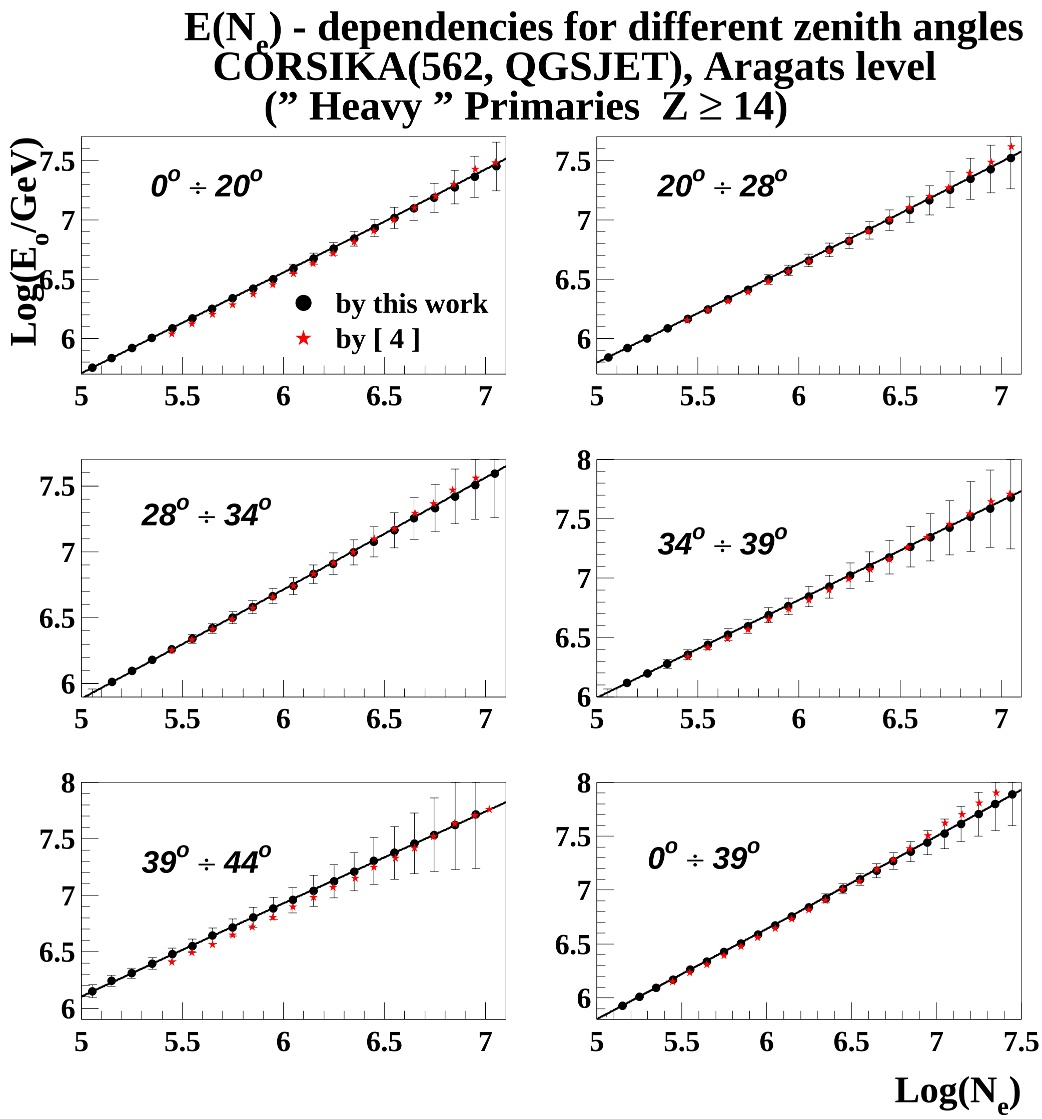


Figure 7:

Figure 2:

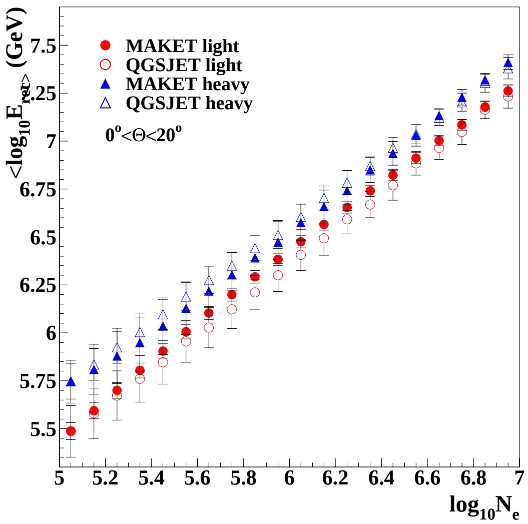


Figure 3: *$E_e st$ versus EAS size for light and heavy nuclei in first zenith angle bin (error bars indicate the RMS deviation of primary energy distribution in N_e bins)*

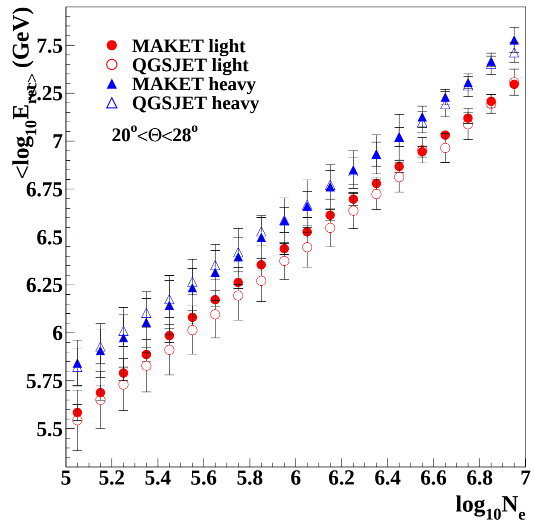


Figure 4: *$E_e st$ versus EAS size for light and heavy nuclei in second zenith angle bin (error bars indicate the RMS deviation of primary energy distribution in N_e bins)*

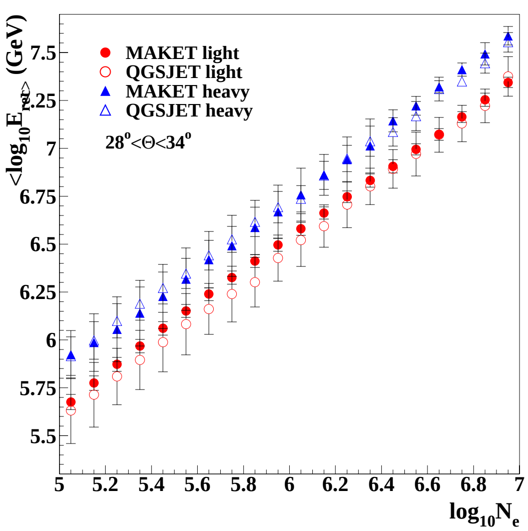


Figure 5: *$E_e st$ versus EAS size for light and heavy nuclei in third zenith angle bin (error bars indicate the RMS deviation of primary energy distribution in N_e bins)*

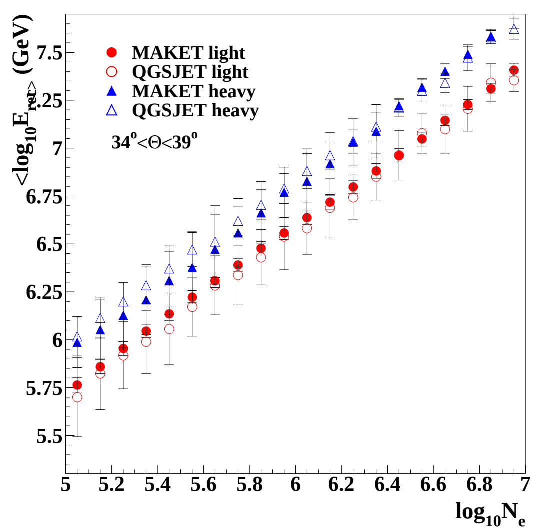
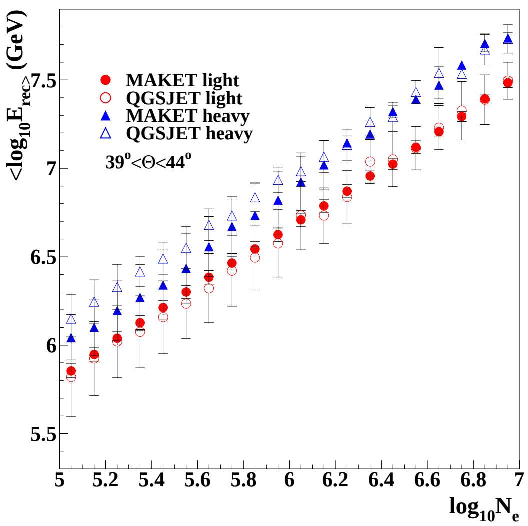


Figure 6: *$E_e st$ versus EAS size for light and heavy nuclei in fourth zenith angle bin (error bars indicate the RMS deviation of primary energy distribution in N_e bins)*



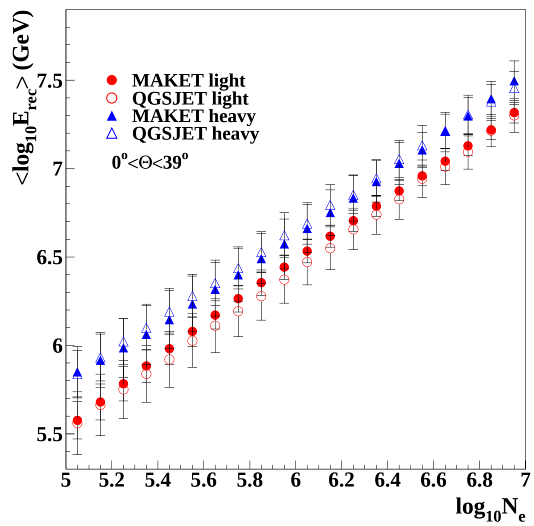
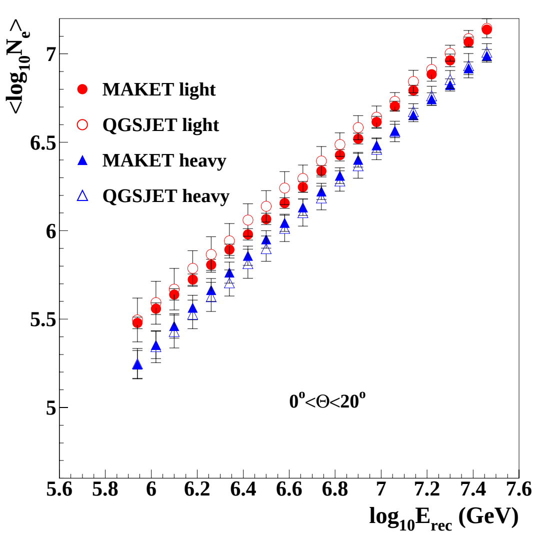


Figure 7: *E_est versus EAS size for light and heavy nuclei in fifth zenith angle bin (error bars indicate the RMS deviation of primary energy distribution in N_e bins)*

Figure 8: *E_est versus EAS size for light and heavy nuclei for all zenith angles (error bars indicate the RMS deviation of primary energy distribution in N_e bins)*



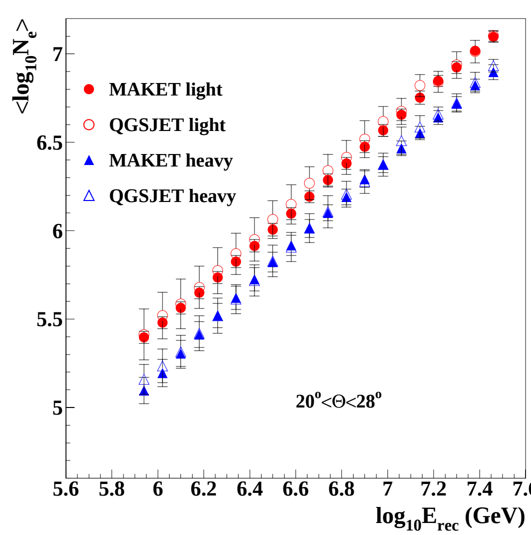


Figure 9: *N_e versus primary energy for light and heavy nuclei in first zenith angle bin*

Figure 10: *N_e versus primary energy for light and heavy nuclei in second zenith angle bin*

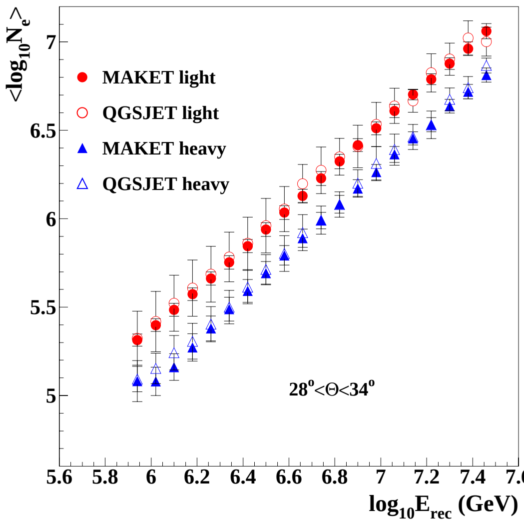


Figure 11: N_e *versus primary energy for light and heavy nuclei in third zenith angle bin*

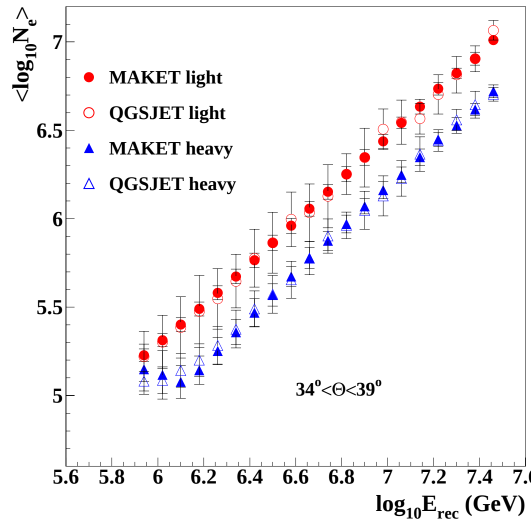


Figure 12: N_e *versus primary energy for light and heavy nuclei in fourth zenith angle bin*

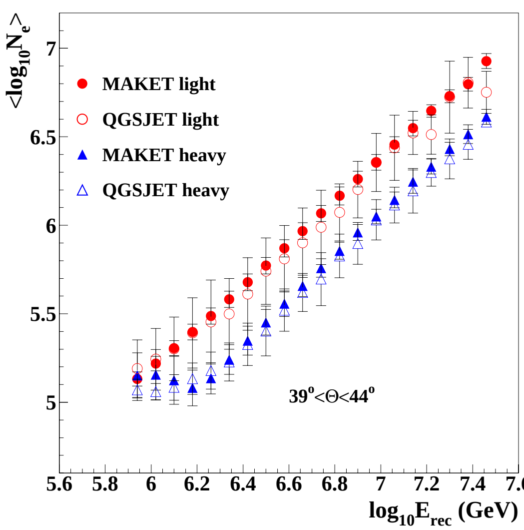


Figure 13: N_e *versus primary energy for light and heavy nuclei in fifth zenith angle bin*

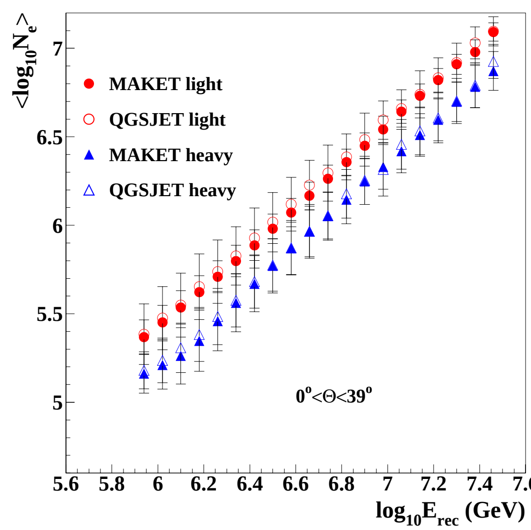


Figure 14: N_e *versus primary energy for light and heavy nuclei for all zenith angles*

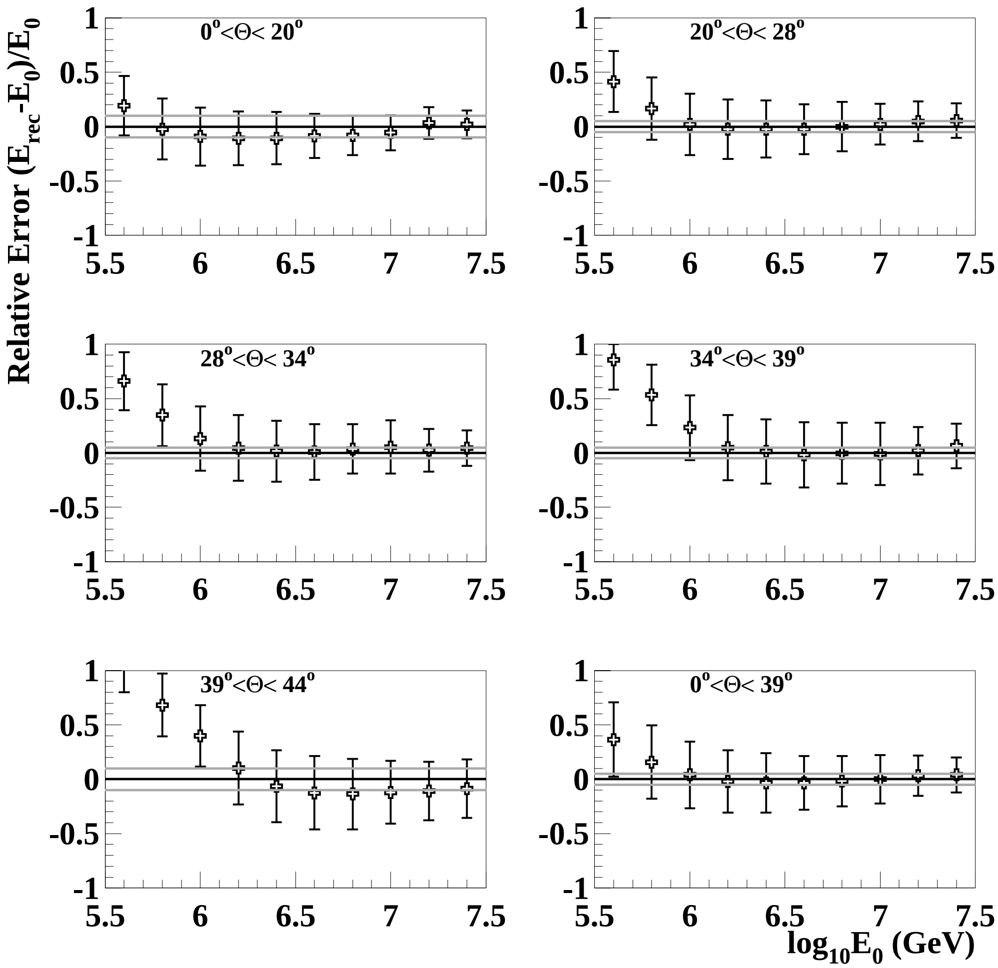


Figure 15: *The relative error of estimation versus primary energy in different θ bins (estimation is performed for whole theta range and results for different bins are extracted)*

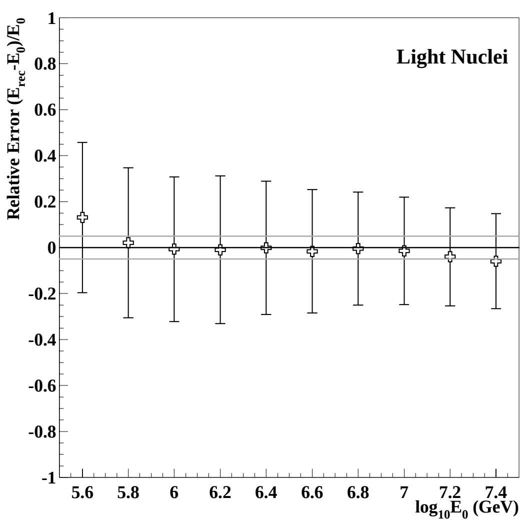


Figure 16: *Primary energy estimation of light nuclei (H,He)*

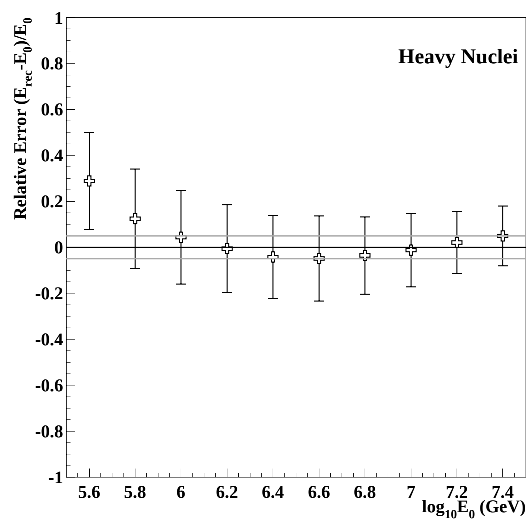


Figure 17: *Primary energy estimation of heavy nuclei (Si,Fe)*

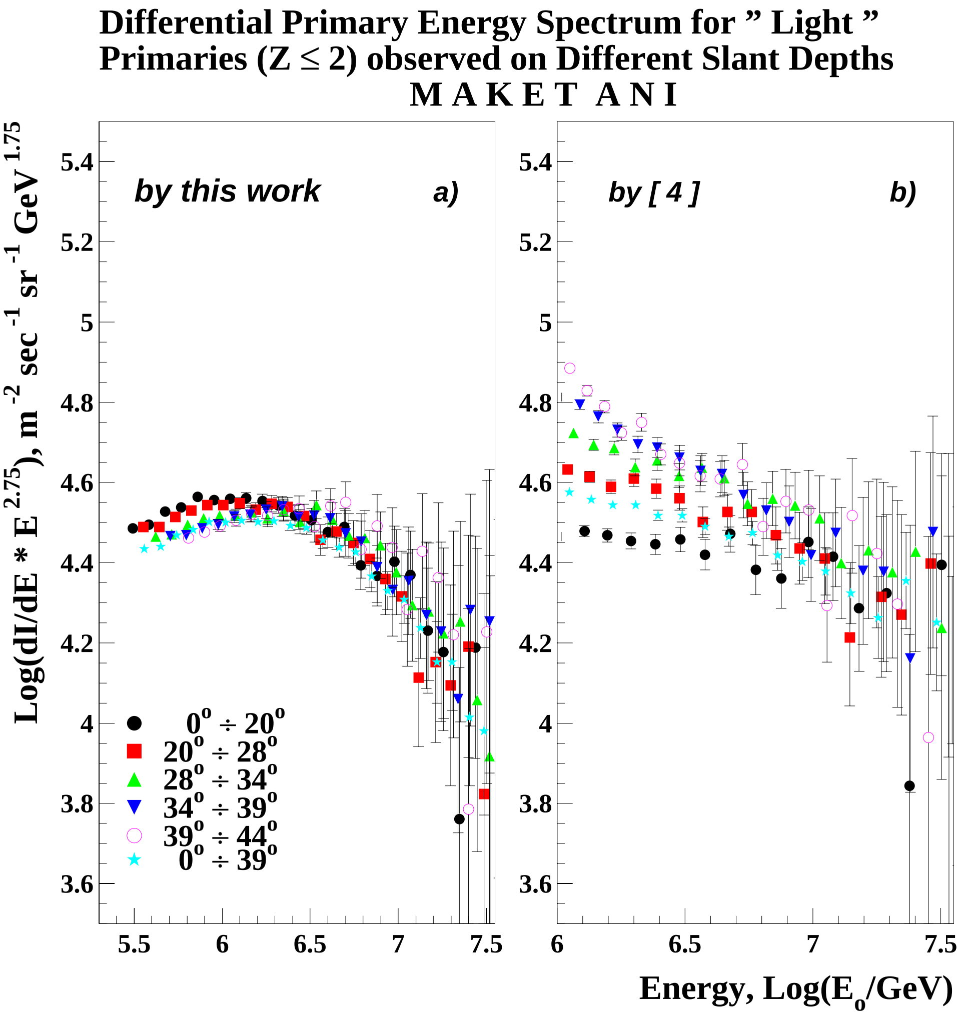


Figure 8:

Figure 18: Comparison of the primary energy spectra for the light group of nuclei obtained by [3] and NN method as presented by Dr. Sokhoyan

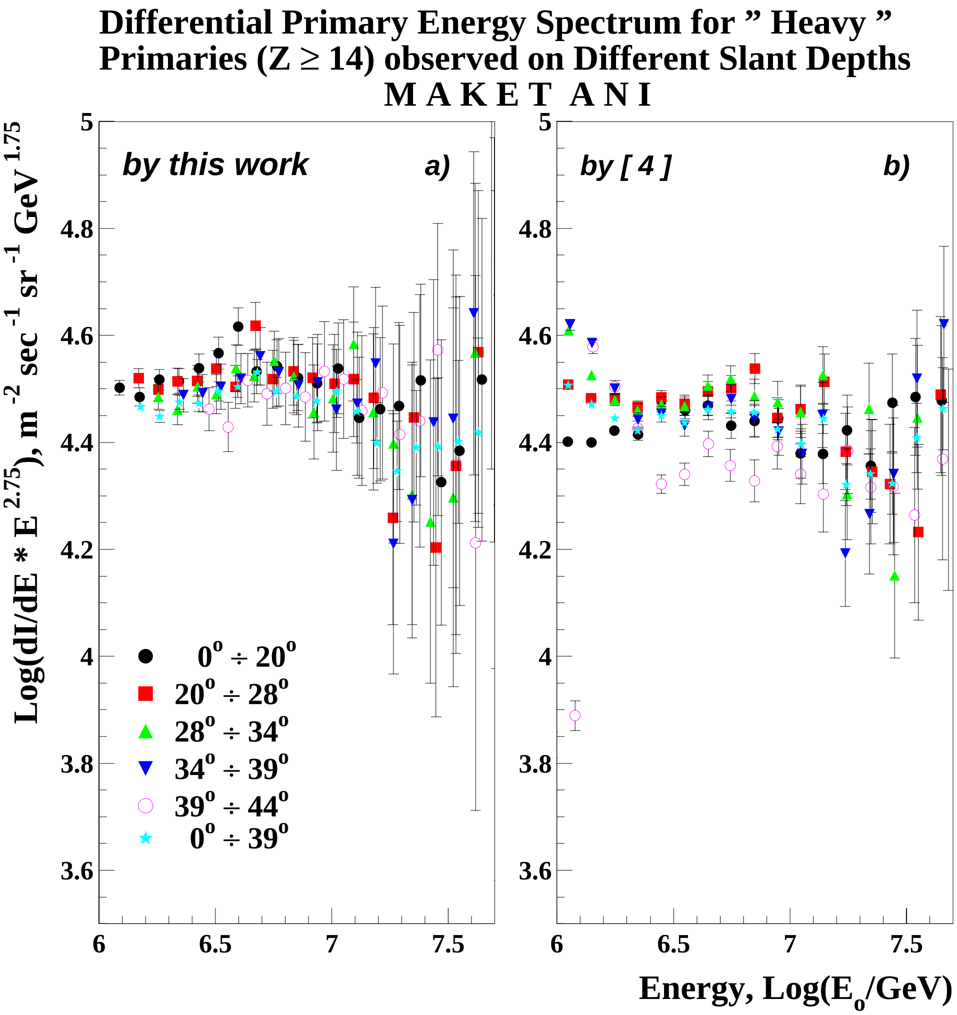


Figure 9:

Figure 19: Comparison of the primary energy spectra for the heavy group of nuclei obtained by [3] and NN method as presented by Dr. Sokhoyan

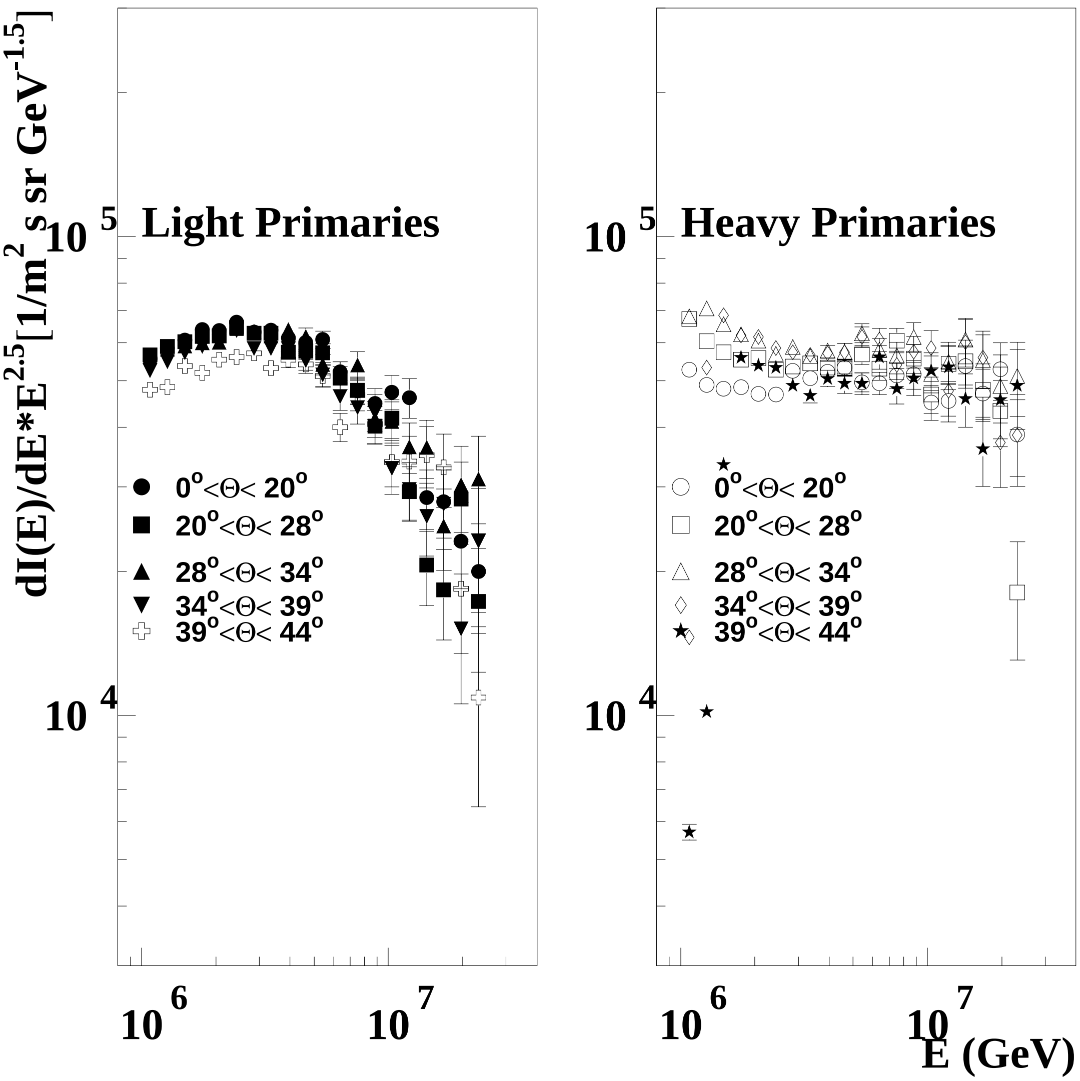


Figure 20: *Primary energy spectra of the light and heavy groups of nuclei for different zenith angle bins (obtained by NN method and constructed by us).*