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Statistical methods for signal estimation of point sources of cosmic rays

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Abstract

The estimation of the significance of the peaks in one- and two-dimensional distributions is one of the most important problems in high-energy physics and astrophysics. The physical inference from low-statistics experiments usually is biased and many discoveries lack further confirmation. One of the typical mistakes in physical inference is the use of non-adequate statistical models. We analyze the significance of the experimental evidence in the on-going efforts of detecting the point source of cosmic rays. We found that simple statistical models (Gaussian or Poisson) did not adequately describe the experimental situation of point source searches. To avoid drawbacks related to usage of the incorrect statistical model, we introduce new extremum statistical models appropriate for the point source searches. The analysis is conducted in the framework of two models utilizing extremum statistics: first – using the fixed grid of celestial coordinates, and second – using the tuned grid (introducing more degrees of freedom in the search). The test distributions for the significance estimation are obtained both from simulation models and from the analytical model of extremum statistics. We show that the second model gives adequate physical inference, while the first model can lead to the positively biased conclusions of the point source significance.

Keywords: Statistical inference; Extremum statistics; Cosmic rays; Point sources

1. Introduction

Searches of the cosmic ray sources is one of the most promising ways to gain insight in the long-standing problem of the origin of these particles. While many experiments have shown that the distribution of arrival directions are isotropic [2 and references therein, 3], existence of the small-angle anisotropies has been claimed by several groups in the "knee" energy region 10^{14}–10^{16} eV [18,7] and the ultrahigh energy range $>10^{19}$ eV [20].

Physicists usually attribute considerably greater than statistically expected positive fluctuation to a "source". However, experience has shown that large excesses, up to 6σ, are more common than were expected [14]. When consistent and reliable statistical tests are applied we cannot obtain convincing proof for point sources. It was demonstrated in 1973 that the evidence for many of the claimed γ-ray sources, when properly treated, is rather weak [16]. Another striking illustration of the importance of accurately assessing the significance of peaks embedded in the low statistics, high background experiments is the "discovery" of the so-called pentaquark particle, which contains four quarks and one antiquark, according to the claims. In 2003 physicists from many laboratories around the world made headlines, announcing that they had found a new particle. There were above 10 particle detections reported with very high confidence level of 5 and even 6σ. Unfortunately, new experiments with better statistics do not confirm existence of the new particle. The "overwhelming body of negative evidence" indicates that the pentaquark might be an artifact [19].

Therefore, in the search of the point sources or new particles, the most important is to prove that observed excess is not background fluctuation only or systematic effect introduced by the detector. Positive excess of counts is compared with mean value of background count rate and

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its variance. It is also necessary to take into account the number of attempts physicist made to reveal the signal more effectively. Any re-binning and shifting of the grid, superimposed on the data, changes the statistical model used for estimating the significance of the source [16]. It is often very difficult to account on all specific experimental procedures applying for the revealing signal. Therefore, the significance obtained using inadequate assumptions usually lead to positively biased significances and observations supported by significances which are unlikely to be chance fluctuations, have not been able to be verified in later experiments. It is due to the overall problem related to the choice of the appropriate statistical model. The ascribed physical inference could be valid within a chosen model, but tell nothing about the validity of the model itself.

In this publication we use both analytical approach and Monte-Carlo method to obtain the statistical model, adequately describing the signal searches. The sources for possible erroneous physical inference based on biased models are clearly stated and discussed. As an example for comparing the different statistical models we consider the observations of the Monogem Ring (MR) by surface particle detector arrays [7]. The MR is a supernova remnant (SNR), located at a distance of ~300 parsec from the Sun, with an ~100K year-old radio pulsar, PSR B0656+14, near the center [21]. Recently, three new observations of MR were published. Two of them [15,6] confirm a signal from MR, while the third one [2] report no signal from MR with very large significance. Nonetheless, the MR continues to be considered as a candidate source for cosmic rays [12,13]. Therefore, we consider the rigorous clarification of the point source search methodology as a very up-to-date and important point, which can help to solve the long-standing problem of the cosmic ray (CR) origin.

2. Monogem Ring observation by the MAKET-ANI detector

In 2003 we reported significant excess of Extensive Air Showers (EAS) whose arrival direction pointed to the Monogem Ring [7]. In the search for the source of the cosmic rays (CR), we used data from the MAKET-ANI detector on Mt. Aragats in Armenia [4,8], from years 1997 to 2003. In the experiment we measure the horizontal coordinates of the incident primary particle, by calculating the axes of the Extensive Air Shower (EAS), namely, zenith angle θ and azimuth ϕ, and then transforming them to equatorial coordinates – Right Ascension (RA) and declination δ, according to the transformation equations [17]. To do this we need to know the angle formed by the detector axes with respect to the direction to the North Pole and the time of event registration, in addition to the horizontal coordinates. After measuring the geographic alignment of the MAKET-ANI array in the summer of 2004, we found an error in the conversion of the measured EAS directions from the horizontal coordinates to equatorial (celestial) coordinates [9], which significantly altered our original conclusion.

As can be seen in Fig. 1, the actual angle between the MAKET-ANI detector axis and the North–South geographic axis of the earth is 17°, while in the MAKET-ANI data base, formed in 2001, zero degrees was assumed. This erroneous assumption resulted in an apparent excess of showers in the histogram bin pointed to the MR direction (signal bin with 43 EAS pointed on the MR direction) as is seen in (Fig. 2(a)). After the correction of the event's equatorial coordinates the excess of the points in the "signal" bin reduced as presented in Fig. 2(b) (only 28 EAS remain). The "migration" of points from the "signal" bin to the neighboring bins is demonstrated in Fig. 3. Compact cluster of showers in the signal bin shown in Fig. 3 as dia-

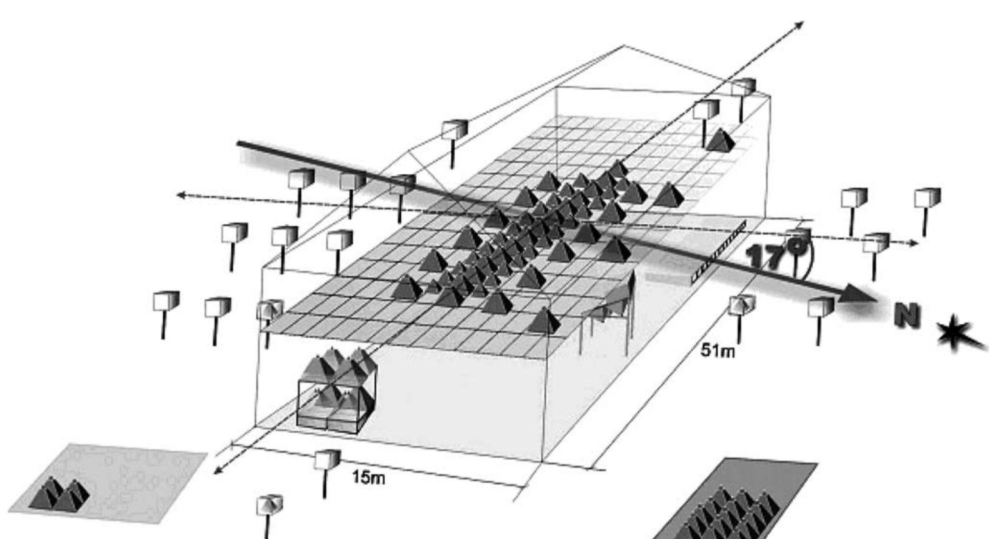

Fig. 1. MAKET-ANI detector, Aragats, Armenia.

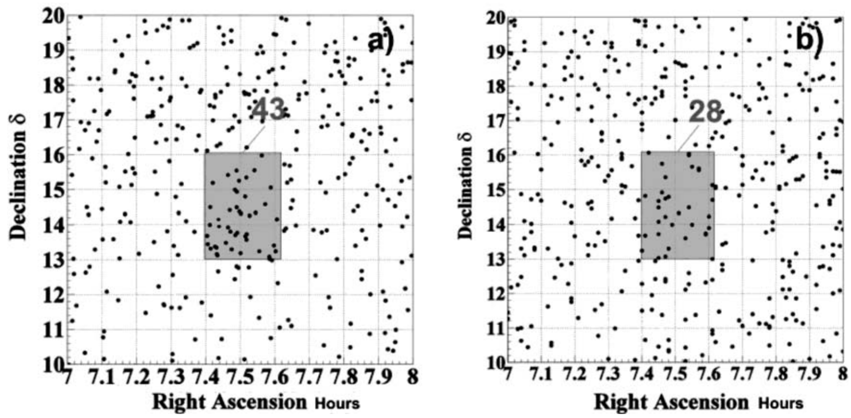

Fig. 2. A part of the sky map (the "signal" bin) obtained from MAKET-ANI EAS data, before (a) and after (b) the correction of the coordinate conversion. Each point in the map represent shower coming from the definite direction.

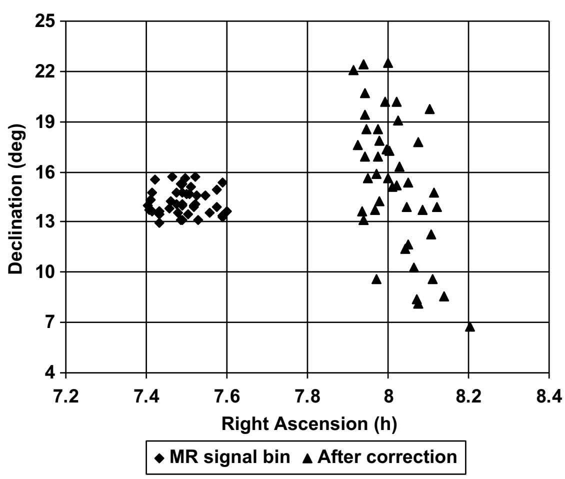

Fig. 3. Migration of events as measured by the MAKET-ANI detector from the "signal" bin to neighboring bins after correction of celestial coordinates.

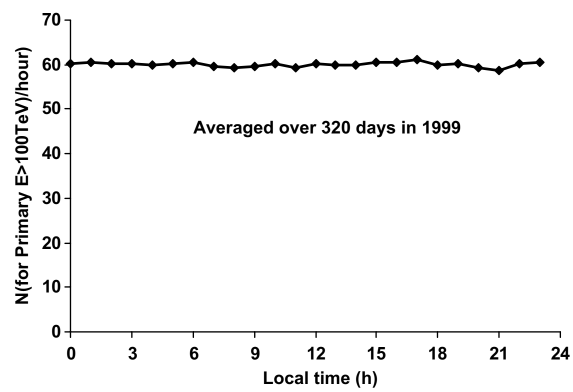

Fig. 4. Averaged daily distribution of MAKET-ANI detector triggers for EAS initiated by primary particles with energies $E > 10^{14}$ eV shows no time dependence.

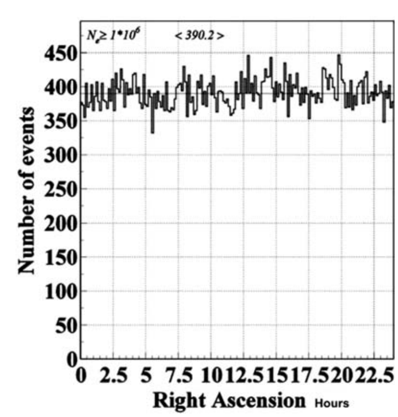

Fig. 5. Distribution of MAKET-ANI EAS data by Right Ascension (RA) for the year 1999.

monds is redistributed among 12 another same size bins shown in same figure as triangles.

By examining the MAKET-ANI EAS database, we found that the detector does not introduce any bias in time, because there was no preferable time or season of particle detection. Fig. 4 shows that the time distribution of the detected particles averaged over 320 days in 1999 is uniform, which leads to uniform distribution by the Right Ascension, as shown in Fig. 5. The analogous distributions of other years are also highly uniform.

However, the distribution by zenith angle in horizontal coordinates is highly anisotropic, because of the different effective thickness of atmosphere for each angle, which incident particles have to pass to reach the detector [1]. For the MAKET-ANI EAS data this anisotropy is well described by a $\cos^6\theta$ dependence, so the distribution of detected particles by zenith angle is described by the function $\sin\theta\cos^6\theta$, having maximum at $\theta \sim 22°$ (as shown in Fig. 6).

When we transform the horizontal coordinates into celestial coordinates, the uniform time distribution is transformed to an isotropic RA distribution, and the anisotropic zenith angle distribution is reflected in the form of

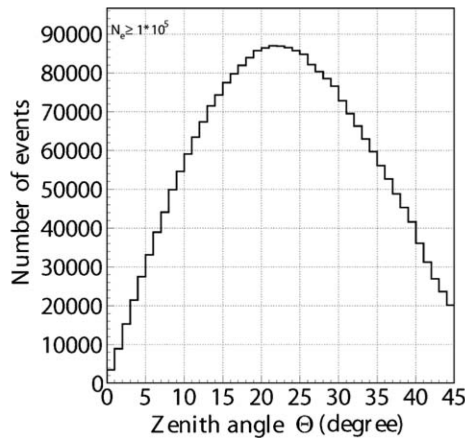

Fig. 6. Distribution of MAKET-ANI EAS data by zenith angle.

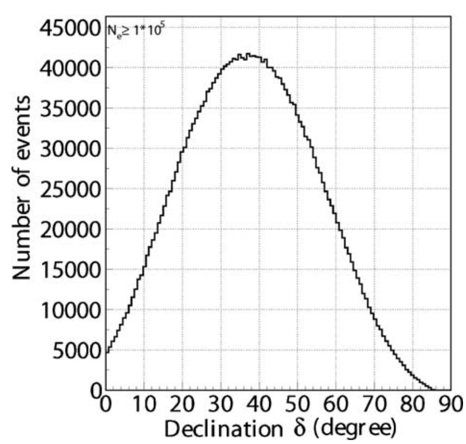

Fig. 7. Distributions of MAKET-ANI EAS data by declination.

anisotropic declination distribution. Since the declination of the zenith is equal to the latitude (40.5° for the Aragats research station), the maximum in the distribution of declinations appears at $\delta \sim 40°$ as shown in Fig. 7. This strong dependence of the number of events upon the declination angle implies that when searching for uniformity in celestial coordinates we have to choose narrow intervals – declination "belts", within which the distribution of events can be treated as approximately uniform.

Proceeding from the detector angular accuracies and available EAS data we choose the bin size as $\Delta\alpha \times \Delta\delta$ $(3° \times 3°)$, covering a $360° \times 60°$ equatorial coordinate range, with 20 declination belts in total, each divided into 120 bins.

The cosmic ray point source should manifest itself as an excess number of counts in one, or several adjacent bins, in comparison with the corresponding belt-average value. In our analysis the Right Ascension Scan (RAS) method [1] was implemented, where not only one, but all declination belts are used to form statistical test distribution. The grid with bin (cell) size of $(3° \times 3°)$ is superimposed on the two-dimensional distribution of the actual values of the celestial coordinates of detected showers, i.e. on the, so-called, sky map. We then examined the distribution of the events within each cell and made further analysis according to the H_0 probabilistic model described below.

As is usual in statistical hypothesis testing, the main hypothesis (H_0) we need to check is in opposition to the hypothesis we are interested in, i.e. we check the hypothesis that the arrival of the CR on the MAKET-ANI detector is isotropic ("no-signal" hypothesis). In this case it means to determine: is the detected enhancement in the "signal bin" a simple random fluctuation of the isotropic distribution? If we see a large deviation of number of events fallen in particular bin from the value expected assuming the validity of H_0, then we will have a very low probability of H_0 being true. Therefore, we can reject H_0. But, of course, it does not imply that the opposite hypothesis is automatically valid. As was mentioned by Astone and D'Agostini [5], a revised version of the classical proof-by-contradiction is hidden in the logic of standard hypothesis testing – "in standard dialectics, one assumes a hypothesis to be true, then looks for a logical consequence which is manifestly false, in order to reject the hypothesis. The 'slight' difference introduced in 'classical' statistical tests is that the false consequence is replaced by an improbable one".

3. Gaussian approximation

The number of events falling in each bin is independently and identically distributed random variables obeying multinomial law. Multinomial process consists of the random realization of one of N_δ possibilities; in our case – classes, representing the division of the range of all declinations into 20 fixed declination "belts". In our probabilistic treatment of the problem we convolute the uniform distribution of RA and treat the number of events hitting different bins as realizations of the multinomial random process with $\bar{N}_j,\ j = 1, 20$ fixed means. Then, by normalizing each bin content by the mean and variance of the corresponding declination belt we obtain standard Gaussian distribution $N(0, 1)$ to be used further as the test statistics:

$$\sigma_{i,j} = \frac{N_{i,j} - \bar{N}_j}{\sqrt{\bar{N}_j}}, \quad i = 1, N_\alpha, 3, \quad j = N_{\delta 1,\delta 2,3} \tag{1}$$

where $N_{i,j}$ is the number of events in the rectangular bins, $\bar{N}_j$ and $\sqrt{\bar{N}_j}$ are the RA averaged mean and mean square deviation of number of events within the bin of jth belt, $N_\alpha = 360$ is the number of RA divisions; $N_{\delta 1} = 6.6$ is the first declination, $N_{\delta 2} = 66.6$ is the last declination for a total 20 declination belts, each of 3°. We were looking for single source candidates in the two-dimensional $\Delta\alpha \times \Delta\delta(3° \times 3°)$ grid, covering a $360° \times 60°$ equatorial coordinate range with $M = 2400$ bins. The rectangular equatorial coordinate system (grid) origin was taken at $(0°, 6.6°)$.

Of course, the multinomial significances are different from the Gaussian ones [11] specially for the large significance values. Nevertheless, first we will present results with the commonly used Gaussian distribution. The bias, introduced due to the slow convergence of Gaussian approximation toward the correct multinomial (Poisson) values, will be discussed in other paper.

More than 2 million showers with sizes starting from $N_e > 10^5$ electrons, detected by the MAKET-ANI detector, were distributed among the $M = 2400$ angular bins. The "signal" was revealed when we examined the sub-sample of $\sim 6 \times 10^4$ events with $N_e > 10^6$. According to the logic of hypothesis testing, we calculate the test statistics by applying Eq. (1) to the experimentally detected showers and using a fixed equatorial grid. As we can see from Fig. 8, the shape of the cumulative distribution of the particles was very close to standard Gaussian distribution N(0,1), with a χ^2 test value of ~1.5 per degree of freedom. Only one direction from the 2400 demonstrates significant deviation from standard Gaussian distribution $N(0,1)$. Therefore, we concluded, that the obtained distribution supports the model of isotropic "background" and "signal" mixed with "background" in one of 2400 equatorial bins.

From the obtained value of 6.04 in the "signal bin", as noted by a circle in Fig. 8, we calculated the corresponding probability of obtaining this value under H_0 hypothesis.

We assumed that maximal obtained value for the signal in bin 6.04 belongs to the $N(0,1)$ distribution. Based on this assumption, the probability density distribution function of obtaining this value as the maximal value among M possibilities is straightforward [10]:

$$P_M(x) = M \cdot g(x)(1 - G_{>x})^{M-1} \quad (2)$$

where $g(x)$ is standard Gaussian probability density for the signal bin; $G_{>x} = \int_x^\infty g(t)\mathrm{d}t$ is the so-called test statistics p-value: the probability to obtain the value of the test statistics in the interval greater than x.

To obtain the probability of observing number of events equivalent to or more than 6.04 standard deviations in one out of 2400 bins (meaning the p-value of the distribution

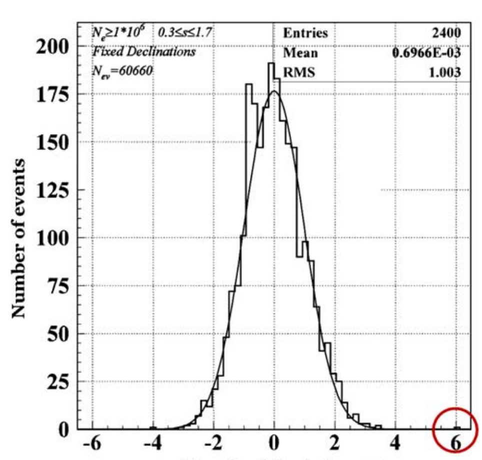

Fig. 8. Signal significance test with full equatorial coverage with 2400, $3° \times 3°$ bins; $N_e > 10^6$, before correction of coordinates conversion. Note the 6.04σ point in the circle.

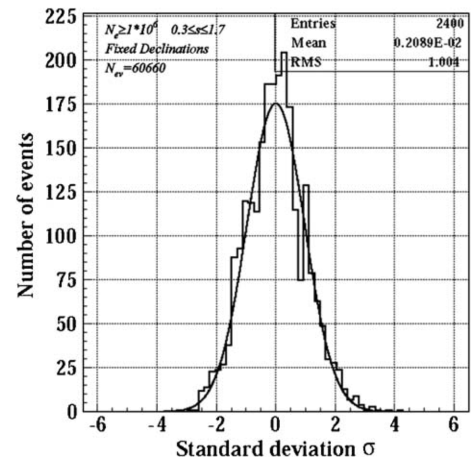

Fig. 9. Signal significance test with full equatorial coverage with 2400, $3° \times 3°$ bins; $N_e > 10^6$, after correction of coordinates conversion. Note that event at 6.04σ has disappeared.

$P_M(x)$), we need to integrate $P_M(x)$ in the interval [6.04, $+\infty$). For $M = 2400$ we obtain $\int_{6.04}^\infty P_M(x)\mathrm{d}x \sim 2 \times 10^{-6}$. Proceeding from this very small value, we rejected the null hypothesis and concluded that the MAKET-ANI has detected signal from the direction of the Monogem Ring.

After correcting the error in the transformation of the MAKET-ANI geographic coordinates to celestial coordinates we found no significant deviation from H_0, as it is seen from Fig. 9. Note that the events at 6.04σ, which existed before the coordinate system correction in Fig. 8, have now disappeared in Fig. 9.

However, the question arises: how did we obtain such low chance probability. Can we explain it as a simple random coincidence, or was it due to the wrong statistical model? We found it very improbable that we were so unlucky that a chance probability of two out of a million was realized. Therefore, we put the statistical model itself under question. In the next sections we will analyze the sources of our error and will develop new methodology for analysis, which will give adequate inference.

4. Bin regrouping effects

The statistical model we use for estimating the chance probability, is dependent not only on the chosen distribution function, but also on the methods of grouping of experimental data. "Where one is looking for deviation from uniformity in a continuum, one cannot escape from the multiplicity of possible groupings" [14]. Usually the physicists adjust the grid superimposed on the sky map slightly, to include the "signal" events in the selected bin as much as possible. The logic of such an adjustment is the following: if a randomly chosen, fixed grid divides the signal between neighboring bins, why not try to shift the grid to contain the entire signal in one bin?

In this logic one random grid is changed to another and it seems that nothing essential happens in the process, but as we will see, such a simple operation dramatically changes the estimate of chance probability. Each regrouping leads to the increase of M in Eq. (2), thus changing the chance probability. Usually, the regrouping effect is not taken into account. Physicists make conclusions on the signal significance by calculating the chance probability according to a simple Gaussian model, often obtaining positively biased significances. To demonstrate that the extremum statistics (2) accounts for the re-binning and provides correct chance probabilities we developed two numerical models.

Our first model generates the random Gaussian variables in 120 RA bins in each of the 20 declination belts according to the belt-specific means and variances as obtained in the experiment, thus generating random sky maps, analogous to that shown in Fig. 2, but for the entire sky seen by the MAKET detector. After applying the normalizing transformation (1) to the generated random map we obtained $M = 2400$ random variables distributed according to the standard Gaussian $N(0,1)$. Then the maximum positive deviation from the $N(0,1)$ was stored as the value of the test statistics.

Our second model generates a number of events in the same way as the first one. Then the origin of the equatorial coordinate system (right ascension and declination) is shifted by 0.1° in the range equal to one bin size (3° × 3°). Thus, instead of one grid $30 \times 30 = 900$ different

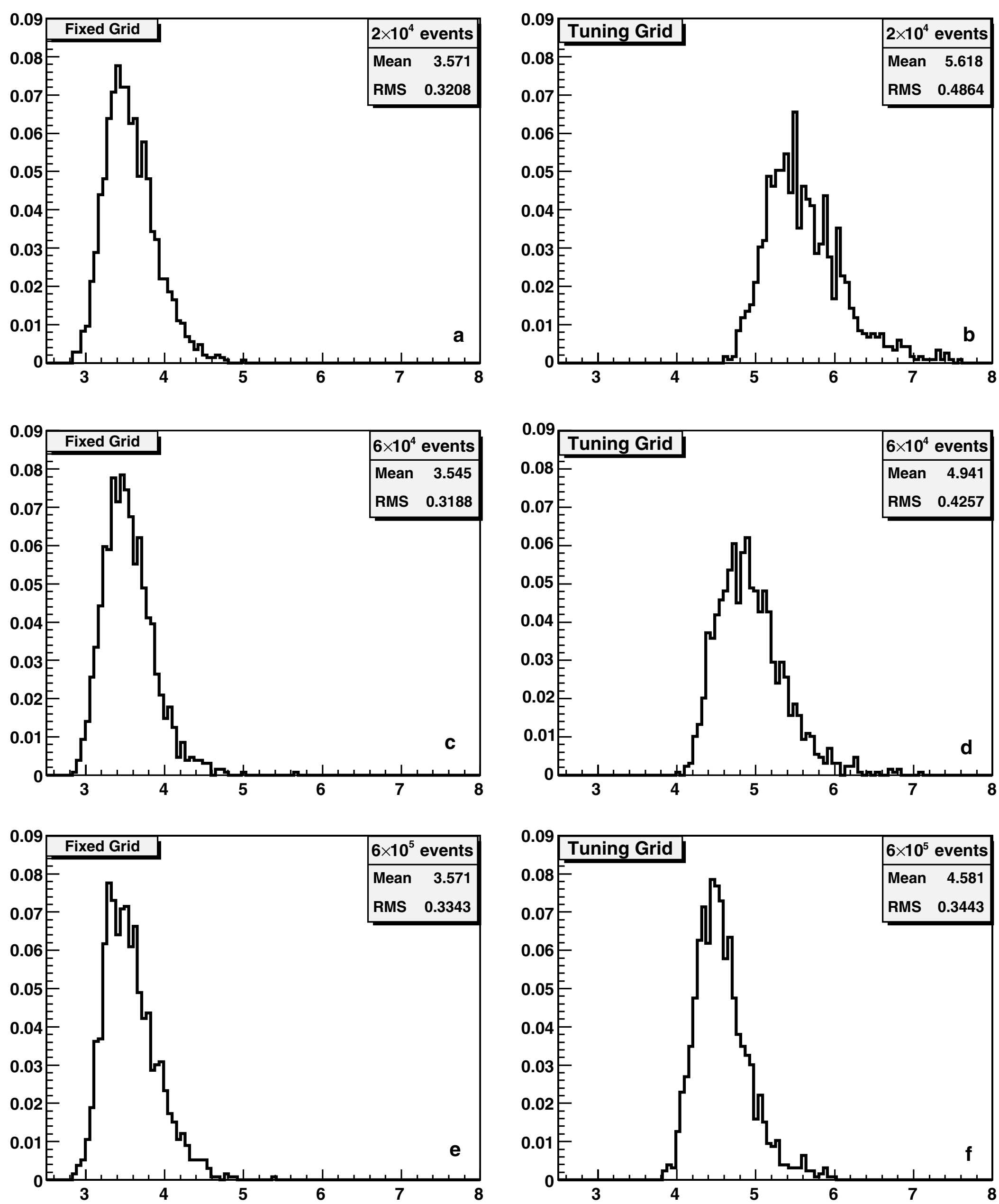

Fig. 10. Distribution of the extremum test statistics values for the first simulation model – one fixed grid with $M = 2400$ bins – (a), (c), (e), and for second simulation model – with tuning the grid to contain maximal signal – (b), (d), (f).

grids are tested. The goal of this procedure is to obtain maximum possible "signal" for given distribution of events. After the shift, the current value of maximum is compared with the previous best one, and if the new one is larger, it is saved as the best. The largest-obtained value of maximum is stored. This value is just the largest positive deviation for the given sky map and grid size obtained in the grid tuning procedure.

The fixed grid model is usually assumed when calculating chance probabilities; nonetheless the tuned grid model describes realistically the experimental situation in peak searching. Our numerical experiment is designed to illustrate how the chance probabilities are changed and why we can obtain very high significances if there is no signal at all.

We generate the random sky map 1288 times, to obtain the distribution of extremum statistics for fixed and tuned grids. As we can see from Fig. 10 the large significance values (large σ) occurred much more often for the tuned grids (Fig. 10(b), (d) and (f)), as compared to the fixed grid model Fig. 10(a), (c) and (e). Therefore, when calculating chance probabilities we are at risk to make optimistically biased inference: to get much higher significance than experiment allows. We also can see in Fig. 10, that by changing the number of experimental points (showers) filling the grid, the "σ" distribution for the tuned grid changes dramatically. For the number of events 2×10^4 the mean of "σ" distribution equals 5.6, while for 6×10^5 events it is 4.5. This demonstrates that if the number of events is small, and bin-to-bin differences are large, then via tuning it is possible to find the combination of event numbers which correspond to very rare fluctuation. When the number of events is enlarged, the corresponding bin-to-bin differences became smaller and it is much more difficult to find large fluctuations.

Therefore, in low-statistics experiments it is possible to find "fake" signal with very large significance. In Fig. 11 we demonstrate how we can obtain a realistic chance probability for the MAKET-ANI experiment. We perform numerical simulations of the MAKET-ANI's detection of the Monogem Ring with both fixed and tuned grid statistical models.

By the solid line in Fig. 11(a) and (b) we denoted the analytical curve obtained from Eq. (2) for the $M = 2400$ (Fig. 11(a)), and $M = 2400 * 600$ (Fig. 11(b)). The histogram on the same figures are obtained with simulations with fixed (Fig. 11(a)) and tuned (Fig. 11(b)) models as described above. The number of events was equal to the one from the MAKET-ANI experiment $\sim 6 \times 10^4$ and 1288 independent random sky maps were generated. From the fixed grid model (Fig. 11(a)) we can see that 6σ (for simplicity we use the 6σ value, instead of 6.04σ obtained in MAKET-ANI experiment) is really very rare fluctuation. The frequency of obtaining 6σ from histogram equals to 0, because we perform only 1288 trials, and, we can expect only ~ 2.5 events from million according to analytical calculations.

The frequency of obtaining 6σ calculated from the "tuned grid" histogram (Fig. 11(b)) equals ~ 2 from hundred. The analytic calculation gives an order of magnitude smaller value compared with frequency obtained from the histogram. However, the difference between the fixed and tined grid models is striking: at least three orders of magnitude!

When we test many grids, probability to obtain large "σ" values is dramatically enlarged. For the MAKET-ANI statistics of $\sim 6 \times 10^4$ events with $N_e > 10^6$, we can easily obtain significance values exceeding 6 and even 7. Therefore, we do not have enough evidence to reject the H_0 hypothesis, if measuring 6.04 value in MAKET-ANI experiment. Remember, that H_0 is the statement that the distribution of cosmic rays is isotropic.

The cause of shift of "σ" distribution mean to larger values when tuning the grid can be explained by enlarging of the number of the tested grids, and, consequently – the number of different bins.

The M multiplier in Eq. (2) represents number of all-possible bins in which the maximum can occur. In the first model we test single fixed grid $M = 2400$. In the second model when we shift the grid, we test M new bins for each

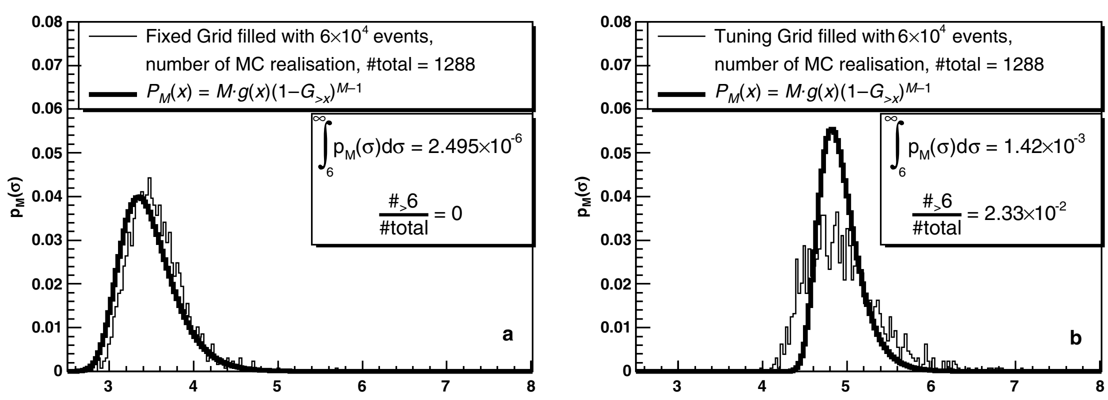

Fig. 11. The comparisons of the two ways of point signal searches, (a) with one fixed grid and (b) with tuned grid to maximize the extremum statistics.

particular shift, enlarging total number of bins to $M * K$, where the K is the number of different grids tested.

Thus the complexity of the second model is $\sim K$ times larger compared to the first one. To check this assumption we enlarge the M value in Eq. (2) till the mode of analytical function (2) comes close to the histogram mode. This occurs at $M = 2400 * 600$, as shown in Fig. 11(b). We reached rather good agreement of the analytic distribution (2) and the tuned grid simulated distribution by the Monte-Carlo method at value of $K \sim 600$, instead of 900 as expected, because not all 900 grids result in different data coverage. Some small shifts leave the distribution of the events in 2400 bins the same. Therefore, such shifts should not be count and value of K is smaller than 900.

5. Conclusions

- In estimating the significance of signal detection, we are looking for the maximum value of deviation of the "signal bin" from the background, and statistical inference is drawn based on the value of this maximum. Therefore, the extremum statistics distribution (2), should be used as the test statistics for estimating the significance of signal.
- Both analytical model (2) and simulated distribution obtained with the tuned grid Monte-Carlo method give very consistent results, proving the necessity to account on all choices of data grouping aimed at revealing the signal.
- Performed statistical analysis of the MAKET-ANI sky maps by the use of the tuned grid model does not support the hypothesis of anisotropy of CR flux; therefore, we withdraw the conclusion of paper [7] which claims the existence of a cosmic ray point source within the Super Nova Remnant Monogem Ring.

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