Preprint YERPHI-1221(7)-90

(EFI-1221-7-90)

ԵՐԵՎԱՆԻ ՖԻԶԻԿԱՅԻ ԻՆՍՏԻՏՈՒՏ
ЕРЕВАНСКИЙ ФИЗИЧЕСКИЙ ИНСТИТУТ
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A STATISTICAL METHOD OF ELEMENTARY PARTICLE MASS DETERMINATION VIA INDIRECT MEASUREMENTS USING SIMULATION DATA

ЦНИИатоминформ
ЕРЕВАН-1990

Նախնատիպ ԵՖԻ-1221(7)-90

Ա.Ա.ՉԻԼԻՆԳԱՐՅԱՆ

ՄՈԴԵԼԱՎՈՐՄԱՆ ԱՐԴՅՈՒՆՔՆԵՐԻ ՕԳՏԱԳՈՐԾՄԱՄԲ՝ ԱՆՈՒՂՂԱԿԻ ՉԱՓՈՒՄՆԵՐԻ ՏԱՐՐԱԿԱՆ ՄԱՍՆԻԿՆԵՐԻ ՔԱՆԳՎԱԾՆԵՐԻ ՈՐՈՇՄԱՆ ՎԻՃԱԿԱԳՐԱԿԱՆ ԵՂԱՆԱԿ

Կարելի է հետևություններ անել բազմամասնիկ ծնման տարբեր ուղիների ներդրման մեծության վերաբերյալ՝ բազմամասնիկ ծնման գործընթացների և ցրանցման գործընթացների մանրամասն մոդելավորման հիման վրա միայն: Մոդելային և փորձարարական տվյալների համեմատության հարցը սկզբունքային նշանակություն ունի և մեծ չափով կանխորոշում է ուսումնասիրվող ֆիզիկական գործընթացների մասին՝ հետևությունների որակը: Ներկա աշխատանքում առաջարկում ենք փնտրվող ֆիզիկական մեծության՝ բազմամասնիկ ծնման փոխազդեցության տարբեր ուղիների հարաբերական մասի, անմիջական գնահատման համար՝ օգտագործել մոդելավորման արդյունքները: Բազմամասնիկ ծնման ուղիների հարաբերական մասի որոշման առաջարկվող [illegible] ... պրոտոնի և նեյտրոնի ... փորձի [illegible] ... մոդելային [illegible] ... [illegible]

Препринт ЕФИ-1221(7)-90

А. А. ЧИЛИНГАРЯН

СТАТИСТИЧЕСКИЙ МЕТОД ОПРЕДЕЛЕНИЯ МАСС ЭЛЕМЕНТАРНЫХ ЧАСТИЦ ПО КОСВЕННЫМ ИЗМЕРЕНИЯМ С ИСПОЛЬЗОВАНИЕМ РЕЗУЛЬТАТОВ МОДЕЛИРОВАНИЯ

Выводы о величине вкладов различных каналов множественного рождения, возможно делать только на основе подробного моделирования процессов множественного рождения и процессов регистрации. Вопрос количественного сравнения модельных и экспериментальных данных имеет принципиальное значение и во многом предопределяет качество выводов об изучаемых физических процессах. В настоящей работе мы предлагаем использовать результаты моделирования для непосредственного оценивания искомой физической величины, относительной доли различных каналов реакции множественного рождения. Предлагаемый метод определения относительной доли каналов множественного рождения был проверен на задаче определения доверительных интервалов на массы суперпартнеров электрона и нейтрино по распадам W бозонов на модельных данных, генерированных согласно условий эксперимента UA1. Показано, что можно различать гипотезы, соответствующие разнице в массе ~10 ГэВ. Созданная методика позволяет вести анализ и в случае измерения многих косвенных признаков (N>>2), когда применение традиционных методов становится проблематичным.

Ереванский физический институт

Ереван 1990

Preprint YERPHI-1221(7)-90

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A STATISTICAL METHOD OF ELEMENTARY PARTICLE MASS DETERMINATION VIA INDIRECT MEASUREMENTS USING SIMULATION DATA

Conclusions on the contribution of different channels of multiple production can be drawn only on the basis of a detailed simulation of the multiple production and registration processes. The subject of a quantitative comparison of simular and experimental data is of a principal importance and in many cases predetermines the quality of the conclusions on the physical processes studied. In this paper we propose to use the results of simulation to estimate directly the physical quantity sought for - the relative fraction of different channels of the multiple production reaction. The proposed method of determination of the relative fraction of the multiple production channels was checked by determining the confidence intervals on the mass of the superpartners of electron and neutrino by the decay of W bosons. The simular data generated under the conditions of the experiment UA1 have been used. It is shown that hypotheses on a mass difference of ≈10GeV can be separated. The technique developed allows to carry out analysis also in case of measurement of many indirect features ($N \gg 2$), when use of traditional methods becomes problematic.

Yerevan Physics Institute

Yerevan 1990

1. INTRODUCTION

A geometric (dimensional) approach to the analysis of multivariate kinematic information was developed in Ref.[1]. The local dimensionality distribution in an N-dimensional feature space (usually a momentum space of final-state particles) allows to draw conclusions on the presence or absence of resonance states. However, the obtained distribution can hardly serve as a basis for determining the fraction of resonance production (relative branching), due to complexity of detection of final-state particles. That is why the corclusions on the contribution of different channels of multiple production can be drawn only on the basis of a detailed simulation of the multiple production and registration processes. The subject of a quantitative comparison of simular and experimental data is of a principal importance and in many cases predetermines the quality of the conclusions on the physical processes studied.

In the present paper we propose to use the results of simulation to estimate directly the physical quality sought for - the relative fraction of different channels of the multiple production reaction.

At such an approach one can estimate the fraction sought for, if even it is impossible to construct the distribution of the effective mass of produced particles, e.g. in case of estimation of the mass of the superpartners by the intermediate boson decays.

2. Search for Superpartners in W Boson Decays

One of the most important tasks of e^+e^-, p^+p^-, e^+p^- colliders is investigation of the W and z boson production and decay. Of special interest are the possible decays to superpartners (SUSY), by the relative fraction of which one can judge about the mass of these hypothetical particles [2-5]. That is why this problem is chosen for simulation. The second reason of choosing this problem is systematic use of simulation in experiments with intermediate bosons, even if their mass can be evaluated only by means of simulation. And finally, the third reason of choosing the problem and the detector UA1 is accessibility of a Monte Carlo program allowing to generate SUSY and SM (the standard model) decays of w bosons at the UA1 array.

In the UA1 experiment w are identified by:

$$p^+ p^- \to W^\pm + X \quad . \qquad (2.1)$$
$$W^\pm \searrow e^\pm + \nu$$

The alternative (SUSY) process is:

$$p^+ p^- \to W^\pm + X \quad . \qquad (2.2)$$
$$W^\pm \searrow \hat{e}^\pm + \tilde{\nu}$$
$$\hat{e}^\pm \searrow e^\pm + \tilde{\gamma} \qquad \tilde{\nu} \searrow \nu + \tilde{\gamma} \; .$$

In both cases electron is the observable. The relative probability of SUSY depends on the mass of the superpartners $\tilde{e}$ and $\tilde{\nu}$; the larger the mass the lower the probability of a SUSY decay. And if we succeed in determining the fraction of SUSY decays (relative to the first, background channel), then we may determine the mass of these particles.

The following transverse mass is measured in the experiment UA1:

$$M_T(e,\nu) = \sqrt{\{2E_T^e E_T^\nu(1 - \cos\Delta\phi^{e\nu})^2\}} \le M_W \ , \qquad (2.3)$$

where E_T^e, E_T^ν are transverse energy of leptons; $\phi^{e\nu}$ is the angle between the leptons on the transverse plane. Note, that when the electron energy is measured directly, then the neutrino energy is recovered from the missing transverse momentum:

$$p_T^\nu = - \vec{p}_T^e - \sum_{i=1}^{K} p_T^i \ , \qquad (2.4)$$

where $\| p_T \| = E^i$ is energy deposited in the i-th cell of the calorimeter, and the direction of the vector P_T^i is defined by the calorimeter impact position. Summation was performed over all cells of the calorimeter, except for cells by which the electron energy had been being determined.

The second value measured was the angle of electron escape to the beam axis. The distribution of this angle for a standard model is described by:

$$dN/d\cos\Theta \sim (1 - q\cos\Theta)^2 \ , \qquad (2.5)$$

where q is the electron charge.

Distributions of the measured values, the transverse mass and the electron escape angle for the standard model are characterized by peaks in the energy range near the mass w (80GeV) and Θ=0 (see Figs.1 and 3). In the region of low energies and small angles there must be fewer background particles. On the other hand, the electrons from decays of SUSY particles, due to kinematic characteristics of the four-particle reaction, have a more uniform angular distribution and lower transverse energy (see Figs.2,4).

These differences, which are clearly seen in Fig.5 by the absence of SM events in the bottom left corner of the scattering diagram, allowed to determine the lower limits on the superpartners mass by the UA1 data.

An additional distinctive feature between SUSY and SM is the fact of a negative correlation between the variables measured in the standard model, whereas there is no correlation in SUSY. The mentioned differences between SUSY and SM give ground for a reliable classification of events into two classes by the Bayesian decision rules, which will be presented in the next section.

3. A Classificational Method of Determination of the Relative Branching Ratio

Let us consider the stochastic mechanism $(A,\mathfrak{P})$ which generates the observation $\vec{v}$ in a multivariate feature space, $\vec{V}=(M_T,\ q\cos\theta)$, A=(SUSY, SM). The invariant probability measure on the basic event space A is given by the total simulation of SUSY, SM decays of W bosons at different assumptions on the mass of superpartners. The set of $\vec{U}$ vectors obtained in simulations is the simular analog of the experimentally measured values of $\vec{V}$. But, as opposed to experimental data, it is known to which of the alternative classes each of the events belongs. These "labeled" events include a priori information about dynamics of the process under investigation, which is given in a nonparametric form, as finite size samples. The sequence $\{U_i t_j\}$, where t_j is the class index, we usually call a training sample (TS) denoted by $(A,\tilde{\mathfrak{P}})$.

Since both, physical processes of particle production and those of registration are stochastic, the data analysis is uncertain in the sense that one need not wait for event separation into compact nonoverlapping groups corresponding to SUSY and SM events. The only thing we can require when classifying experimental data is to minimize the losses due to incorrect classification to some degree and to ensure use of a priori information completely. Such a procedure is the Bayes decision rule with nonparametric estimation of the multivariate probability density function, which, when using a simple loss function (the loss is zero in case of correct classification and is the same at any other error), takes the form:

$$\hat{A} = \eta(v,A,\tilde{\mathfrak{P}})=\mathrm{argmax}\{\hat{P}(A_i/v)\}, \quad i=1,L \qquad (3.1)$$

where $\hat{P}(A_i/v)\sim P_i\hat{P}(v/A_i)$ are a posterior densities, $\hat{P}(v/A_i)$ are conditional densities which are estimated by TS $(A,\tilde{\mathfrak{P}})$ using one of many nonparametric methods available [7], L=2. Initial (a priori) values of P_i are taken equal. The monograph [8] is devoted to the interplay of a priori and experimental information in fraction estimation problems. Here we shall not go into discussion of competence of the choice of a uniform a priori distribution, but only mention that at such a choice the a posterior probability and hence, the results of classification will be totally defined by experimental information, which seems reasonable to us in the given physical task.

To estimate conditional densities, we used Parzen's method with automatic kernel width adaptation [9].

$$P(V/A_i)=1/(2\pi^{n/2}h^n)\sum_{j=1}^{M_i} e^{-r_j^2/h^n}, \quad i=1,L \qquad (3.2)$$

where n is the feature space dimensionality, M_i is the number of vectors of the i-th TS class, r_j is distance to the j-th neighbour in the Mahalanobis metric:

$$r_j=(V-U_j)^T R^{-1}(V-U_j), \qquad j=1,L \qquad (3.3)$$

where R is a sampling covariance matrix calculated by a TS to which U_j belongs, h is the Parzen kernel width.

The classification methods, like all the statistical ones, include the procedure quality test as a necessary element. This stage beside all the others is also necessary for determination of the fraction of SUSY events. The most natural procedure quality estimate is error probability R_M which depends on both the degree of overlapping of alternative multivariate distributions and the decision rule being used (the Bayes decision rules provide minimum R_M as compared to any other one):

$$R_M = E\{\theta[\eta(V,A,\mathfrak{P})]\}, \qquad (3.4)$$

where

$$\theta[\eta(V,A,\mathfrak{P})] = \begin{cases} 0, & \text{at correct classification} \\ 1, & \text{otherwise} \end{cases}$$

The mathematical expectation is taken over all possible samples of volume M and over the whole d-dimensional space of measured values.

Since we do not exactly know to what class the experimental vectors belong, the estimate of R_M we obtain via TS:

$$\hat{R}_M = 1/M_{TS} \sum_{i=1}^{M_{TS}} \theta\{t_j,\eta(U_i,A,\tilde{\mathfrak{P}})\} \qquad (3.5)$$

i.e we classify the $\{U_i\}$ TS and check correctness of classification over the index of the class t_j, j=1,L. However, as numerous investigations have shown (e.g.,[10]), this estimate is systematically biased and hence, a cross-validation estimation is preferable

$$R_M^e = 1/M_{TS} \sum_{i=1}^{M_{TS}} \theta\{t_j, \eta(U_i, A, \tilde{\mathfrak{P}}_{(i)})\} , \qquad (3.6)$$

where $A, \tilde{\mathfrak{P}}_{(i)}$ is a TS with a removed i-th element, which is classified. This estimate is unbiased and has an essentially smaller r.m.s. deviation. Note, that we have the possibility to estimate the probability of various types of errors by imposing to classification various TS classes, $\{U_i, t_j\}$, j=1,L, L is the number of classes.

By R_{ij} we denote the probability of classification of the i-th class events as belonging to the j-th class.

Now let us estimate the a posterior fraction of SUSY events.

It is known [11] that the best estimate of a posterior fraction (in case of uniform a priori information and absence of classification errors) is the empirical fraction

$$P^*_{SUSY} = M_{SUSY}/M_{tot} , \qquad (3.7)$$

where M_{SUSY} is the number of events classified as SUSY events, M_{tot} is the total number of events registered during the experiment. If there are any misclassifications, it can be shown (see [12]) that the a posterior fraction is expressed by:

$$\hat{P}_{SUSY} = \frac{P^*_{SUSY} - R_{SUSY\to SM}}{1 - R_{SUSY\to SM} - R_{SM\to SUSY}} . \qquad (3.8)$$

Note that all the estimates of the classification error probability R and the fraction P^*_{SUSY} calculated according to the experimental data are obtained by the same TS, using the same decision rules. The accuracy of estimates is defined by the TS size and the number of experimental data as well as by the value of the classification errors, which present the "quality" of discrimination in the chosen feature subset. To improve the statistical accuracy of classification, we used the bootstrap method developed in Ref.[13], which allows to obtain the final sample replicas by means of the random choice procedure with replacement and to investigate the statistical characteristics of fraction estimation.

4. Results of Simulation

The purpose of simulation was to illustrate the method when determining the mass of superpartners. For that purpose we were given some values of the mass of $\tilde{e}$ (the mass of $\tilde{\nu}$ was taken zero in all cases), the corresponding training samples were generated, followed by the "experimental" independent control sample with a fraction of SUSY events defined by the mass of $\tilde{e}$. After that the fraction of SUSY events was determined by the technique presented in the previous section.

As is seen from the formula (3.8), for correct recovery of the fraction the classification errors must be small (the total error $R_{TOT}=R_{SUSY}+R_{SM\to SUSY}\ll 1$) and besides, the fraction of SUSY events is desirable to be remarkably different from zero, otherwise there will arise problems when interpreting the "negative fraction".

The difference between the SUSY and SM distributions, distinct correlations between the features measured, lead to quite small errors of classification, $R_{TOT}\approx 0.3$.

To make the fraction of SUSY events somewhat larger, the region of M_T<60GeV was chosen for analysis, and though in this case we lost some part of SUSY events, the (SM) background decreased more remarkably, which led to increasing of the relative fraction of SUSY events.

In the fraction determination we were given the mass of M_τ=20-50GeV, which corresponded to (with account of the M_T cut off) the SUSY fraction from 50% to 20%. The Table shows the values of fraction reconstruction with the corresponding errors obtained by the bootstrap technique.

5. Conclusion

The offered method of determination of the relative fraction of multiple production channels was tested in determining the confidence intervals on the mass of the electron and the neutron superpartners by the W boson decays on simulated data generated under the conditions of the experiment UA1.

It is shown that the hypotheses may be separated by a difference in mass of ≈10GeV.

This technique allows to carry out analysis also in case of measuring many indirect features (N≫2), when the use of traditional methods becomes problematic.

The author is grateful to H.Gulkanian, S.Haywood,F.James and S.H.Matinyan for useful discussions, to S.Haywood also for provision of the program of simulation of W boson decays in the UA1 experiment.

Table

Reconstruction of the Fraction of SUSY Events

$M_{\tilde{e}}$ (Gev)	% SUSY	$\hat{P}$
20	21	21±3.1
30	16	18±2.3
40	10	11±3.1
50	3.1	6.1±3.3

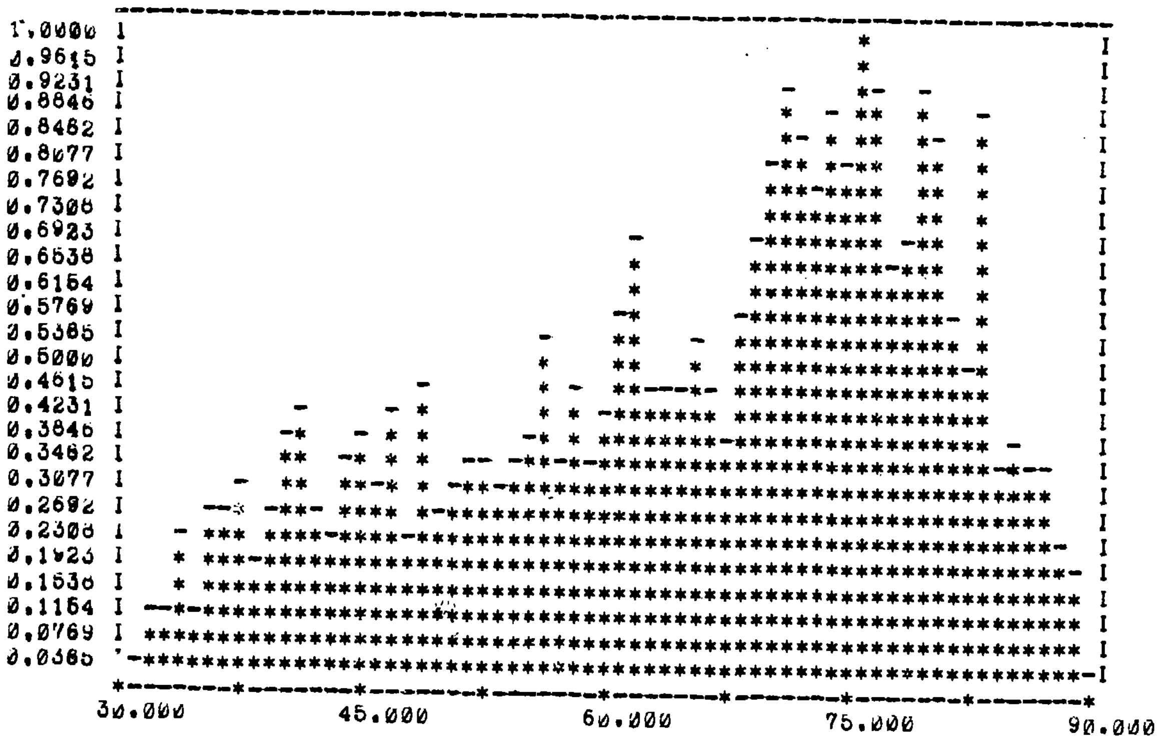

FIG.1 VARIABLE - TRANSVERSE MASS,* - STANDART MODEL,1000 EVENTS

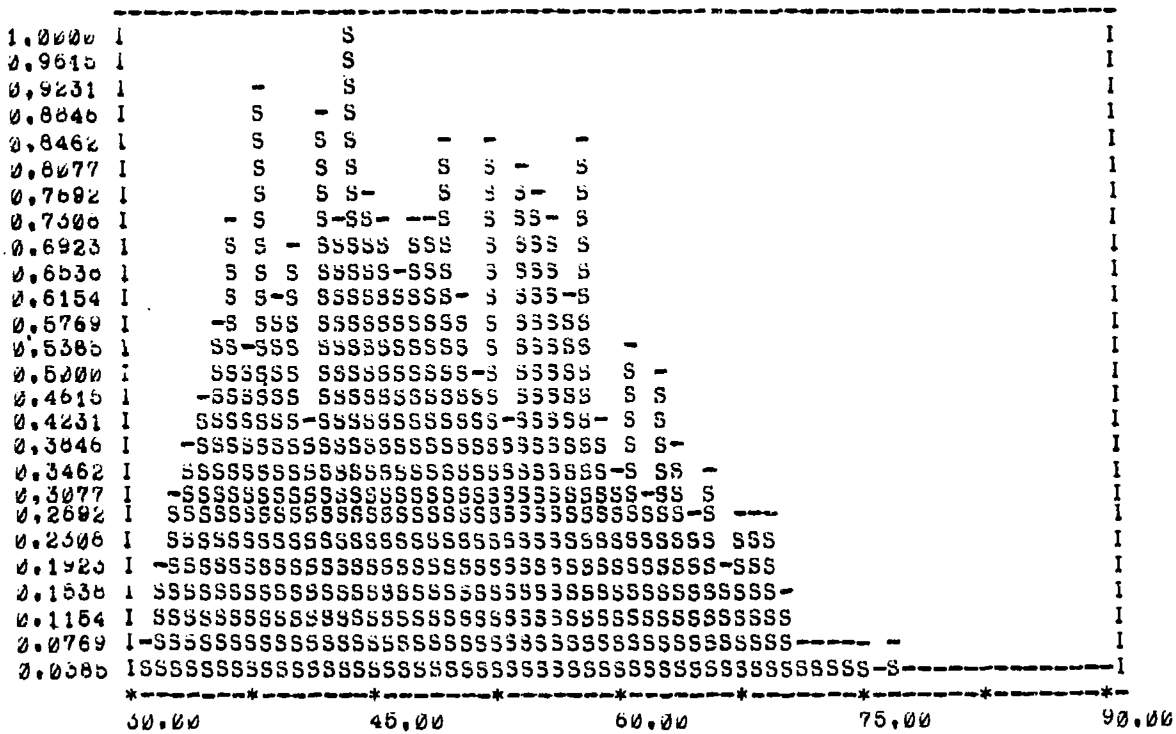

FIG.2 VARIABLE - TRANSVERSE MASS,S - SUSY,1000 EVENTS

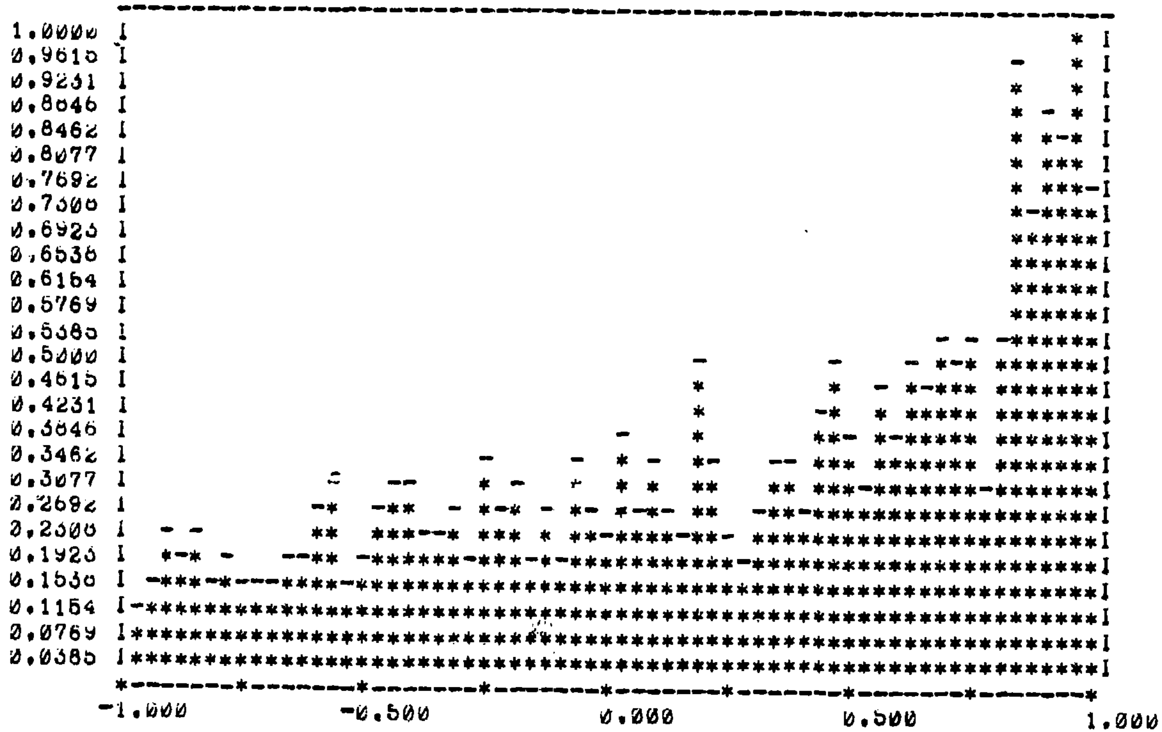

FIG.3 VARIABLE - Q*COS(THETA),* - STANDART MODEL,1000 EVENTS

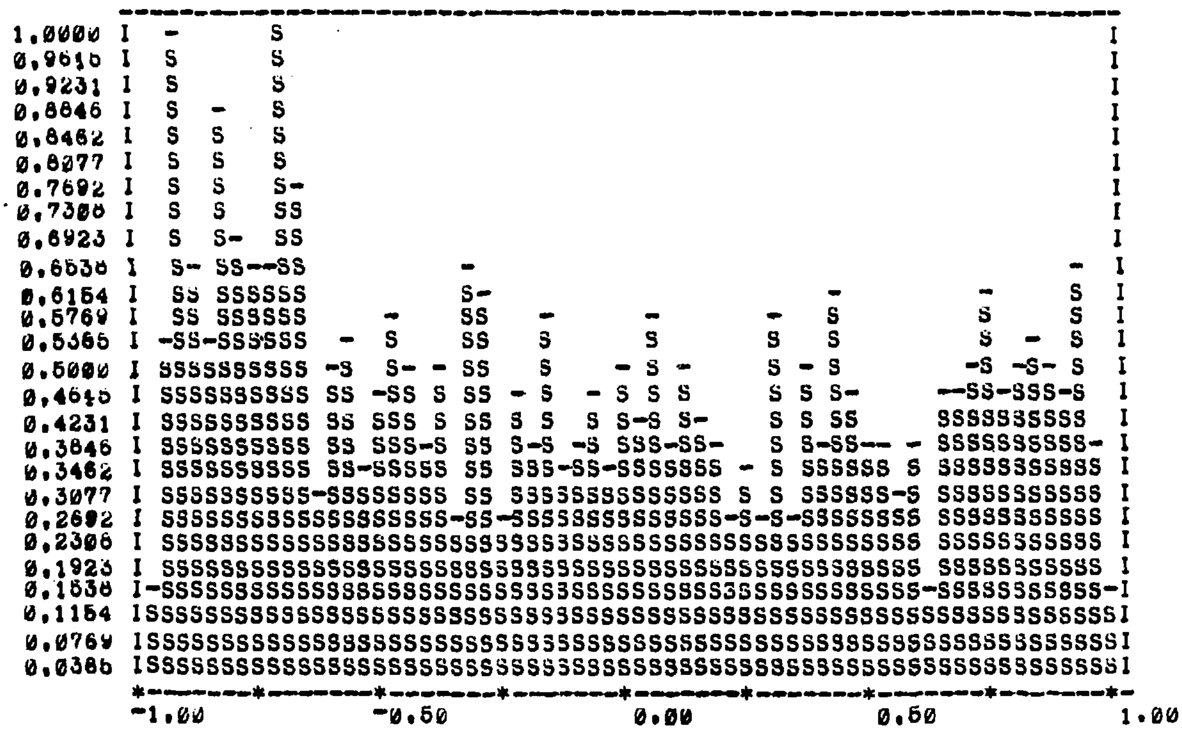

FIG.4 VARIABLE - Q*COS(THETA),S - SUSY,1000 EVENTS

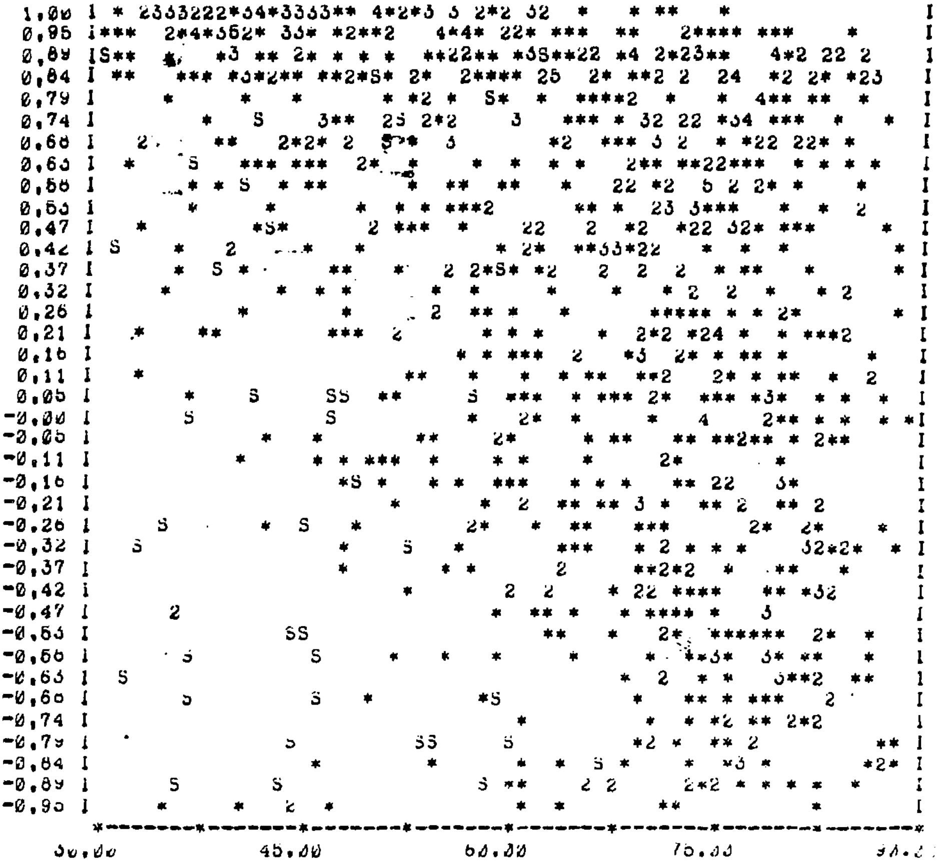

FIG. 5 SCATTER PLOT TRANSVERSE MASS VS Q*COS(THETA)
* - STANDART MODEL, CORRELATION -0.388
S - SUSY, CORRELATION -0.023
RELATIVE BRANCHING RATIO 5%

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The manuscript was received December 26, 1989

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ЧИЛИНГАРЯН

...МЕТОД ОПРЕДЕЛЕНИЯ МАСС ЭЛЕМЕНТАРНЫХ ЧАСТИЦ ПО ...ИЗМЕРЕНИЯМ С ИСПОЛЬЗОВАНИЕМ РЕЗУЛЬТАТОВ МОДЕЛИРОВАНИЯ

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