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A NON-PARAMETRIC METHOD OF ESTIMATION OF THE ENERGY OF γ-QUANTA FROM POINT SOURCES REGISTERED BY MEANS OF CHERENKOV TELESCOPES

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НЕПАРАМЕТРИЧЕСКИЙ МЕТОД ОЦЕНИВАНИЯ ЭНЕРГИИ γ-КВАНТОВ ОТ ТОЧЕЧНЫХ ИСТОЧНИКОВ С ПОМОЩЬЮ ЧЕРЕНКОВСКИХ ТЕЛЕСКОПОВ

Успешное применение техники многомерного анализа для дискриминации Черенковских изображений от фоновых адронов при наблюдении Крабовидной туманности телескопом Уипловской обсерватории, позволившее довести соотношение сигнал/шум до ~50, делает возможной постановку задачи индивидуальной оценки энергии γ-кванта. В настоящей работе показано, что в случае разбиения всего модельного материала на события, попавшие в отдельные зоны - концентрические круги, состоящие из отдельных ячеек фотоприемника, - влиянием прицельного параметра можно пренебречь, то есть снимается зависимость от координат центра Черенковского пятна. Кроме того, мы не использовали заранее выбранную форму функциональной зависимости энергии от измеряемых параметров вспышки. Данные моделирования непосредственно используются в процедуре оценивания, что, как нам кажется, позволяет избежать дополнительных ошибок, связанных с аппроксимацией данных, подверженных сильным случайным искажениям. Полученная относительная средне-квадратичная ошибка, 25-30%, дает надежду на достоверное оценивание энергии γ-квантов от Крабовидной туманности, зарегистрированных обсерваторией Уиппл.

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1. INTRODUCTION

Two circumstances underlie the present work. First, the successful application of the multidimensional analysis technique for discrimination of γ-quanta Cherenkov patterns from background hadrons observed from the Crab Nebula by means of the Whipple observatory telescope. A signal content of ~50% was achieved as compared to less than 1% in raw data [1]. Second, a rather good agreement of the total Cherenkov telescope simulation conducted by different groups with the observational data from the Crab Nebula. At present the Crab Nebula is perhaps the only reliably ascertained source of very high energy γ-quanta.

The first circumstance makes the task of individual γ-quantum energy estimation self-consistent, as we can be sure that half of the events is really initiated by primary γ-quanta. The second circumstance gives us hope that it is competent to utilize both the Cherenkov flash parameters for discrimination from the background [2] and the light intensities in the photoreceiver channels for estimation of the initial energy of γ-quanta.

The first work on numerical estimation of the energy of individual γ-quanta seems to be [3], where the polynomial form of the two parameters, the total Cherenkov light and the flash pattern centre coordinates, is taken as an estimate. The polynomial form coefficients were sought for in two energy ranges separately. The mean-square error turned out within 30-40%.

In the present work it is shown that in case of dividing all the simulation data into events fallen into separate zones - concentric circles consisting of separate photoreceiver cells - one can ignore the impact parameter, i.e. dependence on the Cherenkov spot centre coordinates is removed. Besides, we did not use the prechosen form of functional dependence of energy on the flash parameters measured. The simulation data are used in the estimation procedure, this, in our opinion, allowing to avoid additional errors due to approximation of the events having undergone strong accidental distortions.

The mean-square error obtained to be 25-30%, gives us hope for a reliable estimation of the energy of the γ-quanta from the Crab Nebula registered at the Whipple observatory.

2. BRIEF DESCRIPTION OF THE TELESCOPE AND THE CALCULATION TECHNIQUE

The Whipple observatory Cherenkov telescope is in details described in many works (see, e.g., [4]). We are going to present only some parameters of the new photoreceiver that allows to improve the pattern quality, and hence, the reliability of gamma-ray observations from the Crab Nebula. The new photoreceiver consists of 91 photomultipliers (PM) having a diameter of 2.9cm, and 18 PMs with diameter 5.0cm. The inside 91 PMs are placed in 5 concentric hexagonal circles (zones) numbered 0 to 5 (0 is the central zone, the zone 5 consists of 30 PMs). Angular size of cell is $0.25^0 \times 0.25^0$.

The following rigid event selection criterion is adopted: any 2 of the inside 91 PMs of the photoreceiver must detect a signal exceeding some critical $q_0 > 40$ photoelectrons. The cells

with a signal less than 10 photoelectrons were set to zero [5].

The numerical analysis carried out in the present work is based both on the Monte Carlo simulation of development of EAS initiated by cosmic gamma-rays, protons and nuclei, and on the simulation of registration of Cherenkov radiation from such showers. The calculation algorithms used for this purpose are given in Ref.[6].

The charged component of cosmic rays was assumed to consist of protons (95%) and α-particles (5%). The contribution from heavy nuclei (≤1%) was ignored. It was also assumed, that the α-particle in the first interaction disintegrated into four nucleons with equal energies. To make the calculations easier, they were divided into two stages, the description of which is given below.

In the first stage of calculations, on the basis of the same realizations of the cascade process, there were simulated the responses of a rather large number of Cherenkov γ-telescopes placed at different distances from the shower core. It was assumed that each telescope had a photoreceiver with small rectangular cells ($0.25^0 \times 0.25^0$).

In the second stage there is realized:

a) Passing to the calculation results that correspond to a uniform distribution of the shower core location.

b) Account of the hexagonal structure of the photoreceivers and isotropic cosmic-ray background.

c) Passing to the calculation results that correspond to given energy spectra of γ-radiation and cosmic rays by introducing corresponding statistical weights.

d) Calculation of Cherenkov image parameters used in further analysis. These are the geometrical parameters of the light spot and the light intensity in separate cells of photoreceiver.

3. The Nonparametric Regression Method

Suppose, a flux of particles is sporadically and independently incident on the atmosphere in accordance with some spectrum f(E). Then these particles, undergoing random collisions and interactions with air atom nuclei, initiate an extensive air shower, the parameters of which are registered by the experimental setup, i.e. each value of E is put into coincidence with some random vector of measurements, $\vec{X}$, according to some conditional probability density $P(\vec{X}/E)$.

The peculiarity of solution of the regression problem in the cosmic-ray physics is the fact that neither the true spectrum f(E) nor the conditional density $P(\vec{X}/E)$ are known in the general case, but there is a training sequence $\{E_i, X_i\}$, $i=1,M_{TS}$ (obtained by simulation) and it is required to "recover" the regression $E=E(X)$ by this sequence (M_{TS} is the number of events in the training sample).

In the absence of systematic errors $M\{X\}_E=X(E)$ (the mathematical expectation of random vector measurement at a fixed independent variable (energy) is equal to the regression function value in that point) this problem is reduced to one of minimization of the average risk

$$I(\alpha) = \int (E-F(\vec{X},\alpha))^2 \, P(X,E) \, dX \, dE \,, \qquad (1)$$

where $F(X,\alpha)$ is some functional family depending on the parameter α, $P(X,E) \sim P(X)P(X/E)$ is the probability density function. If there is available a priori information about the form of probability function and the chosen functional family

$F(\vec{X},\alpha)$ is not too complex, then the regression problem can be solved by the least mean squares or the maximal likelihood standard methods.

Due to the complicated stochastic picture of particles and nuclei passing through the atmosphere and the detectors, we have not to expect a standard probability interpretation of all random processes, that is why we have chosen a method based on a nonparametric way of treatment of a priori information, which does not impose any structure and totally uses the information carried by TS.

The method is based on the obvious fact that the events close to some metric (usually the Mahalanobis metric [8] is used) in the feature space have similar energy – the compactness hypothesis. The method based on consideration of the "nearest neighbours" is first analyzed in Ref.[9]. In this work it was shown that when the number of the nearest neighbours, K, and the total number of events in TS, M, tend to infinity so that $K/M \to 0$, then the risk of the procedure tends to the minimum achievable Bayes risk and even the use of one neighbour increases the risk only twice as compared to the Bayes risk. The uniform consistency of the following estimate is shown in Ref.[10]:

$$\hat{F}(X) = \sum_{i=1}^{k} C_i E_{[i]}(X), \qquad \sum_{i=1}^{k} C_i = 1, \qquad (2)$$

where $E_{[i]}(X)$ is the value of the independent variable (energy) of the i-th nearest neighbour of the event $\vec{X}$ in the feature space.

Despite the fact that the nonparametric procedures are optimal under unlimited sampling, for the case of finite samples there are practically no theoretical and practical recommendations on the choice of the method parameters (e.g., the number of nearest neighbours).

For best parameter value selection were simultaneously calculated several estimates corresponding to different method parameters k = 3,5,7,9,11. The weights were taken inverse proportional to the distance from the estimated event from its's nearest neighbour.

4. THE γ-QUANTA ENERGY ESTIMATION

Still Hillas and Patterson have mentioned that the radial distribution of the Cherenkov light from the EAS initiated by gamma-rays (with respect to the centre of the field of view of the telescope) is quite stable [11], which is confirmed by our calculations (see Fig.1). Fig.1 shows the maximum intensity in a cell vs. the impact parameter for the zone 3 events (the zone number was identified by the location of the cell with maximal intensity). As it is seen from the correlation matrix calculated over the events fallen into zone 3 (Table 1), the correlation of the two highest intensities AMP1 and AMP2 with RR impact parameter are negligible. On the other hand, very strong correlation of these parameters with initial energy E_0 indicates linear dependence of quantity of light on the initial energy (see Fig.2), this giving a reason to estimate energy only by intensity of light.

The results of energy estimation by model sampling are presented in Fig.3. The estimation was carried out in the space of two maximal tube intensities AMP1, AMP2, the number of the nearest neighbors used was 7, the obtained half width of distribution of relative errors of estimation was 0.25.

5. CONCLUSION

There is proposed a simple method of estimation of the energy of gamma-rays registered by Cherenkov telescopes. The method employs the very strong correlation of intensity of light with the initial energy in case of separate zone-by-zone event analysis. The high estimation accuracy (25-30%) together with the technique of multidimensional γ-ray event selection allows us not only to investigate the spectra of discrete sources in the energy range of 0.1 to 10 TeV, but also to carry out investigations of interaction of very high energy "γ-beams" with the atmospheric target.

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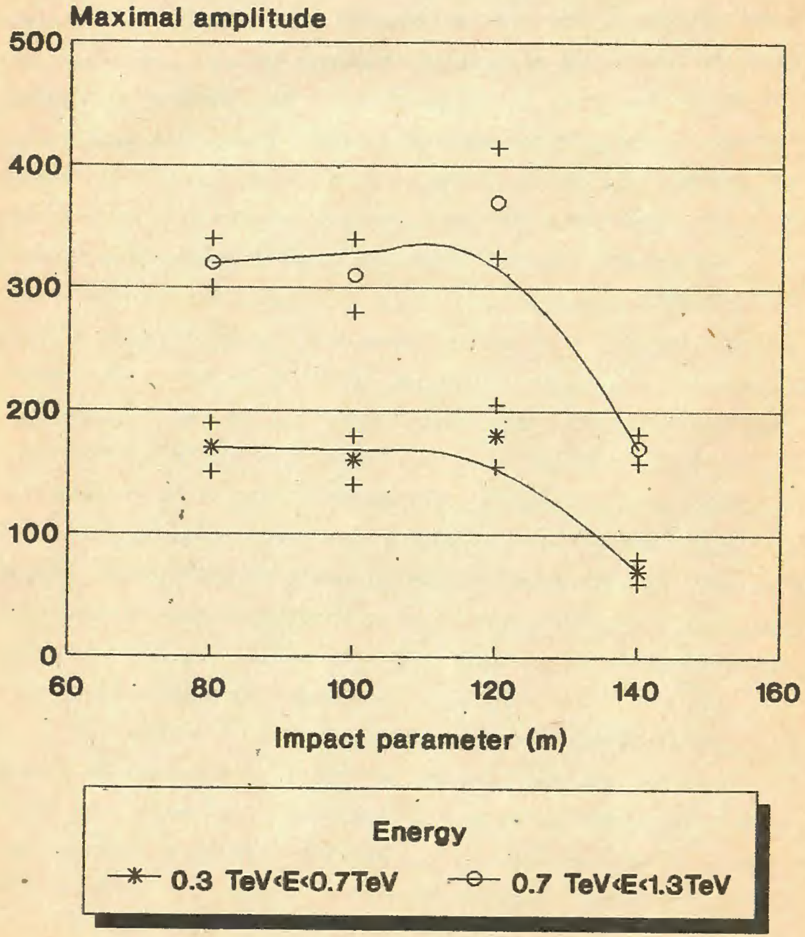
Fig.1 Maximal tube amplitude dependence
for Whipple observatory telescope.



Simulation data

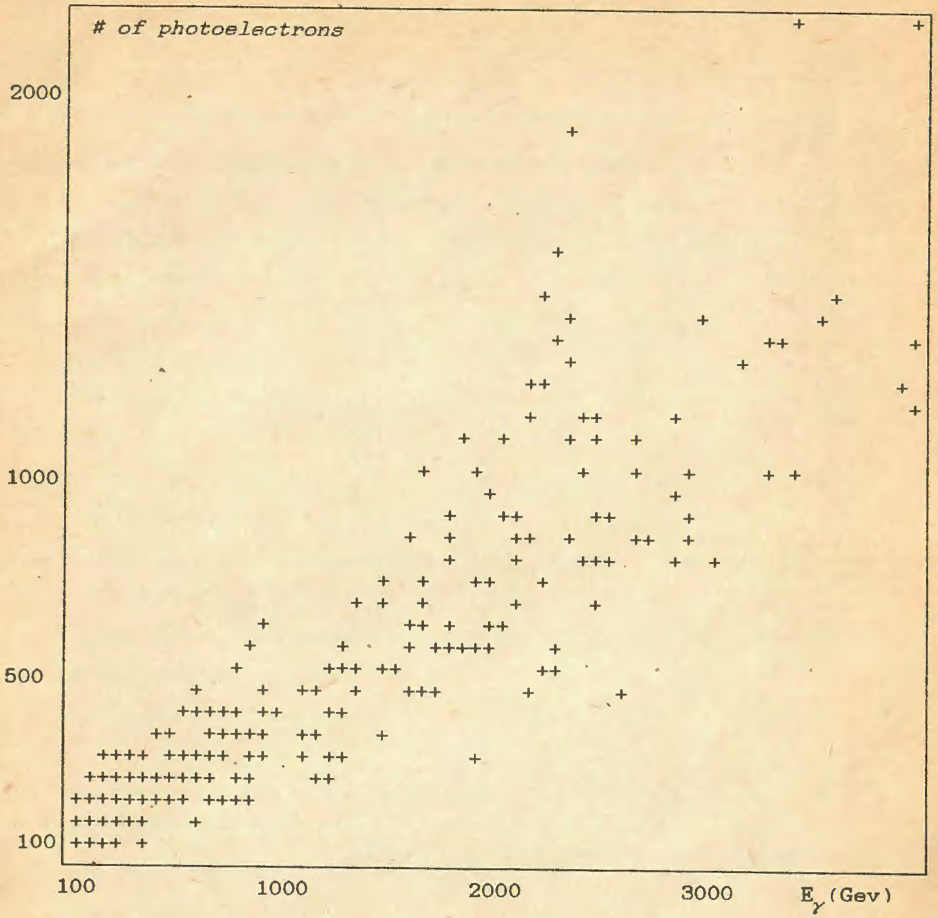


Fig.2 Scatter plot: γ-quanta energy vs phototube maximal amplitud
450 events. correlation - 0.927

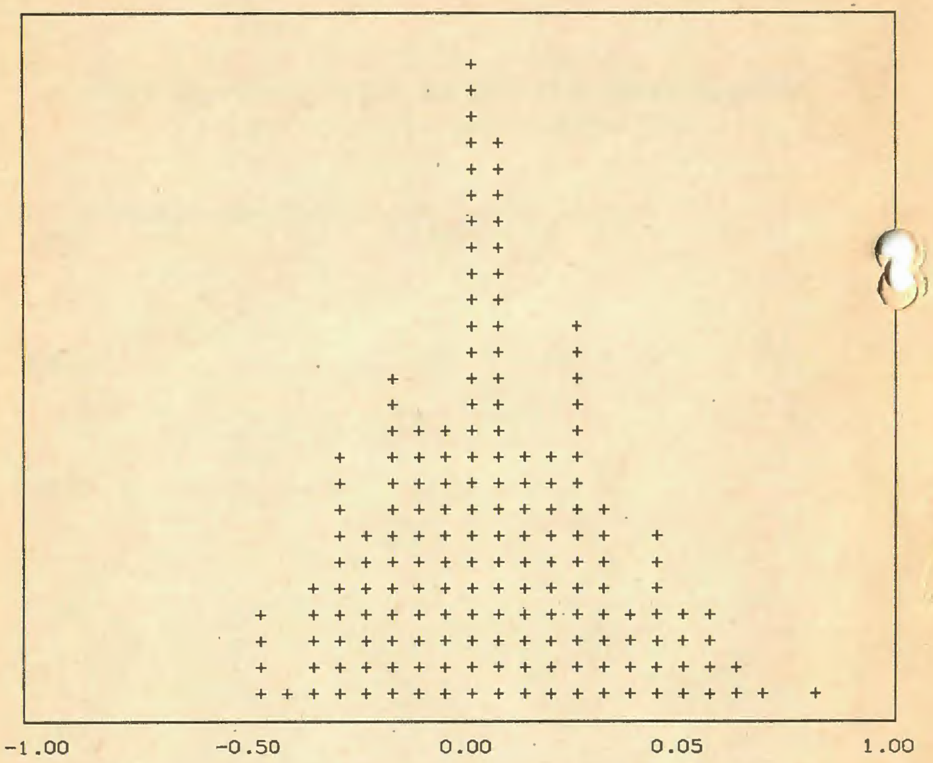


Fig.3 Histogram of energy estimates relative error,
number of events - 450,
number of nearest neighbours - 7,
M.S.D. ~0.25

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НЕПАРАМЕТРИЧЕСКИЙ МЕТОД ОЦЕНИВАНИЯ ЭНЕРГИИ γ-КВАНТОВ
ОТ ТОЧЕЧНЫХ ИСТОЧНИКОВ С ПОМОЩЬЮ ЧЕРЕНКОВСКИХ ТЕЛЕСКОПОВ
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